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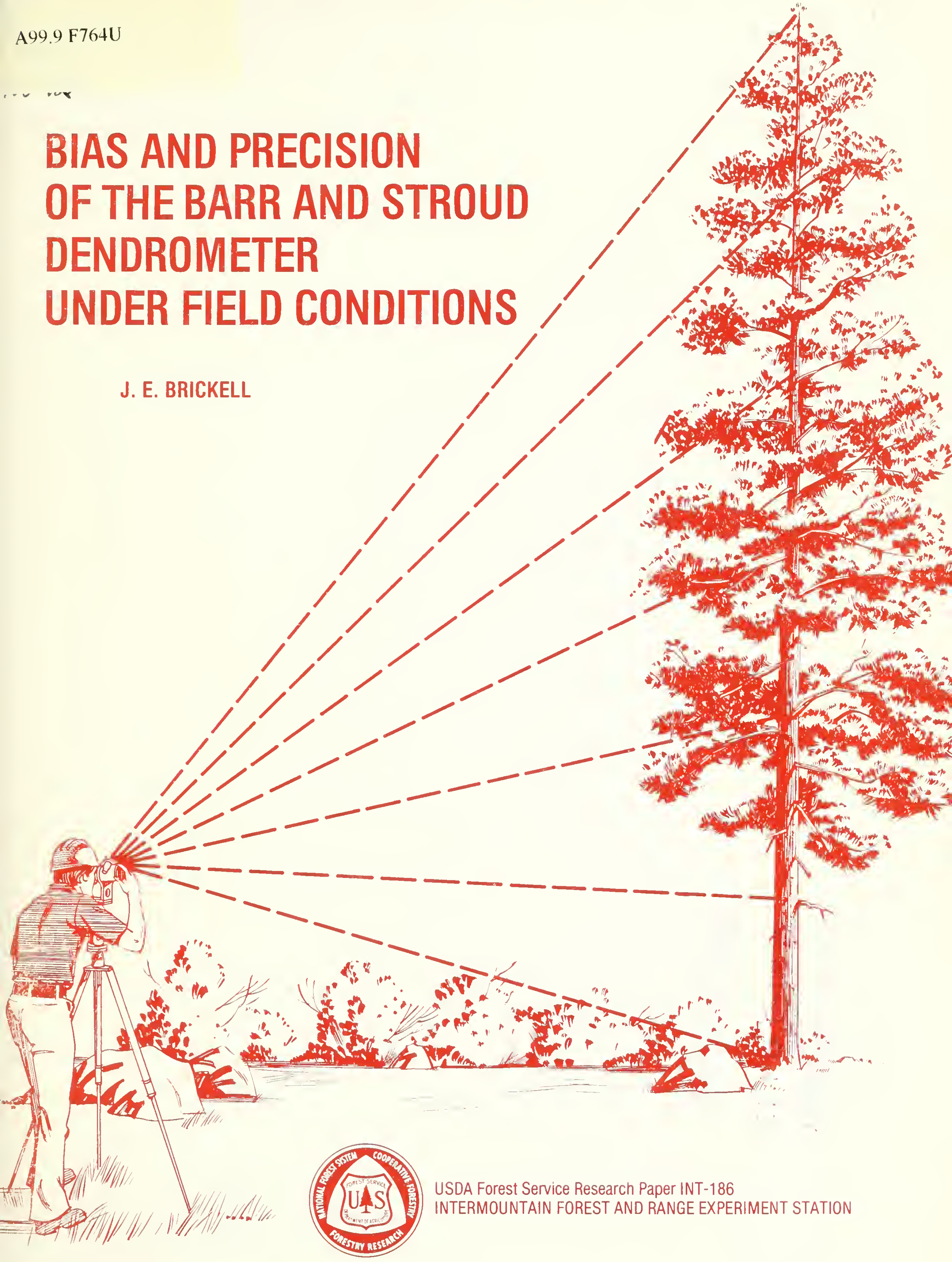
J. E. BRICKELL



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ABSTRACT

Field tests of the Barr and Stroud optical dendrometer disclosed a 1.19 percent bias in tree diameter measurements for each of three observers. No bias in height measurement was discovered. A 5.4 percent bias in standing tree volume measurement was attributed partly to diameter measurement bias and partly to wide spacing of measurement points on tree stems. Advantages and disadvantages of the Barr and Stroud dendrometer compared to other means of tree measurement are discussed.

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INTRODUCTION

One of the problems encountered in the beginning of scientific forest management is still with us, the problem of estimating with accuracy and precision[1] standing timber volumes. This estimation process can be divided into two parts (Cunia 1965):

1. Obtaining an adequate and representative sample of trees from the population for which the volume estimate is sought, and

2. Measuring the volume of sample trees accurately and precisely.

The sample may consist of one tree or a small number of trees, or it may consist of a complete enumeration. Regardless of sampling design, no sample will yield a satisfactory estimate of timber volume unless volumes of individual trees can be obtained within the desired limits of accuracy. This estimate can be obtained directly or indirectly.

Direct volume measurement is an attempt to derive the actual volume of a sample tree from a series of measurements taken on that tree, usually by using a simple geometric solid as a model for segments of the tree stem. Tree volumes computed from a series of stem measurements on standing trees are direct measurements. Direct measurement also includes measuring the volume of water displaced when the tree stem is immersed in a tank.

Indirect volume measurement also uses tree measurements with some predicting device such as a volume table that predicts the average volume of trees having the same dimensions, usually only d.b.h. and height. Volume tables, which include predicting equations and alinement charts, have been the most commonly used means of tree volume estimation since 1780.

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[1] In statistical terminology, accuracy means lack of bias, or the degree to which sample estimates represent the population parameters being estimated. Precision refers to the dispersion of estimates from repeated samples of the same size around their own average. The two terms do not mean the same thing.

Volume tables have also been misused. Proper use of volume tables usually requires that they be checked against a sample of trees measured by direct methods and adjusted if necessary. Most volume tables are not based on a sample drawn from the populations to which the tables will be applied. There may be no reason to suppose that the populations are greatly different, but biased volume estimates might be obtained if the assumption of no difference is not tested. Construction of a local volume table requires direct measurements of tree volume unless it is based on a standard volume table known to be applicable. If a tarif table system is employed, direct measurements of tree volume may be necessary to choose the correct tarif (Turnbull and Hoyer 1965).

Volume tables usually predict volumes only for undeformed, single-stemmed trees; so estimates may be reliable for well-managed stands where there are few abnormal trees. Where injured and forked trees are frequently found in unmanaged forests, the total volume estimate might be biased through procedures used to adjust tabular volumes for abnormal trees. Grosenbaugh (1964a, 1964b, 1965, 1967a, 1967b) devised the 3-P sampling technique, which can circumvent the use of volume tables and rely upon a sample of direct tree volume measurements as a sample of volume on a forest area. Properly used, the sampling method results in much less bias than could be expected from use of many volume tables.

Direct measurements of tree volume are usually obtained by measuring felled trees. Many foresters are reluctant to fell trees that may not be utilized, especially valuable species or crop trees under rotation age (Winters 1930). In addition to the loss of volume and growing stock, leaving felled trees provides breeding places for certain insects and increases fire hazard in the forest. Also, trees measured may not be the property of the party or agency conducting the survey. In other cases, such trees might be located on permanent sample plots that cannot be destroyed.

Climbing trees to obtain direct measurements also has certain disadvantages. Spurs injure cambium and provide infection courts for disease entry. Ladders long enough to be of use are too unwieldy and heavy to carry through a forest. Non-injurious climbing devices require a bole free of branches, yet the branches that prohibit their use generally are too small to support a man's weight. If branches are strong enough for climbing, hard shoe soles can injure them. These difficulties make tree climbing expensive and potentially unsafe.

Naturally, foresters have for many years wanted an instrument that would provide diameter measurements on standing trees with satisfactory accuracy. The Barr and Stroud dendrometer, manufactured by Barr and Stroud Ltd. of Glasgow, Scotland, appears to be the best optical instrument currently available for this purpose. It is also the most expensive.

The study reported here was to evaluate the Barr and Stroud dendrometer's accuracy and precision in a test situation and under actual field conditions. Dendrometer measurements taken under test conditions are compared with mechanical caliper measurements. Tree volumes computed from dendrometer measurements in the field are compared with volumes computed after the trees were felled. Dendrometer measurements of diameter at breast height and total tree height are compared with measurements taken with a tape. Finally, the utility of the Barr and Stroud dendrometer for various tree measurement objectives is compared with the utility of felled-tree measurements and those taken with other optical instruments.

LITERATURE REVIEW

Early Dendrometers

Dendrometers to measure standing trees have existed for nearly 200 years (Fernow 1907; Prodan 1965). At least eight different dendrometers were available in Europe at the beginning of this century (Clark 1902; Graves 1906).

Between 1920 and 1950, dendrometer inventions appeared periodically in the literature (Lönroth 1926; Schumacker 1926; Winters 1930; Hartman 1949). Bitterlich's relaskop was a development of the 1950's and 1960's (Bitterlich 1952, 1958, 1962; Daniel and Sutter 1955; Sutter 1965). McClure (1969) independently invented a mirror caliper similar to that of Heikkila (1932). For a more complete review of previous concepts and instruments see the papers of Hummel (1951) and Grosenbaugh (1963).

Optical Rangefinder Dendrometers

Dendrometers Similar to the Zeiss Teletop

Apparently, the Zeiss Teletop (Ruden 1939; Prodan 1965; Mesavage 1969b) was the first optical rangefinder dendrometer with an optical rangefinder of variable base length. An unmodified Teletop is shown in figure 1. Fricke (1967) modified the

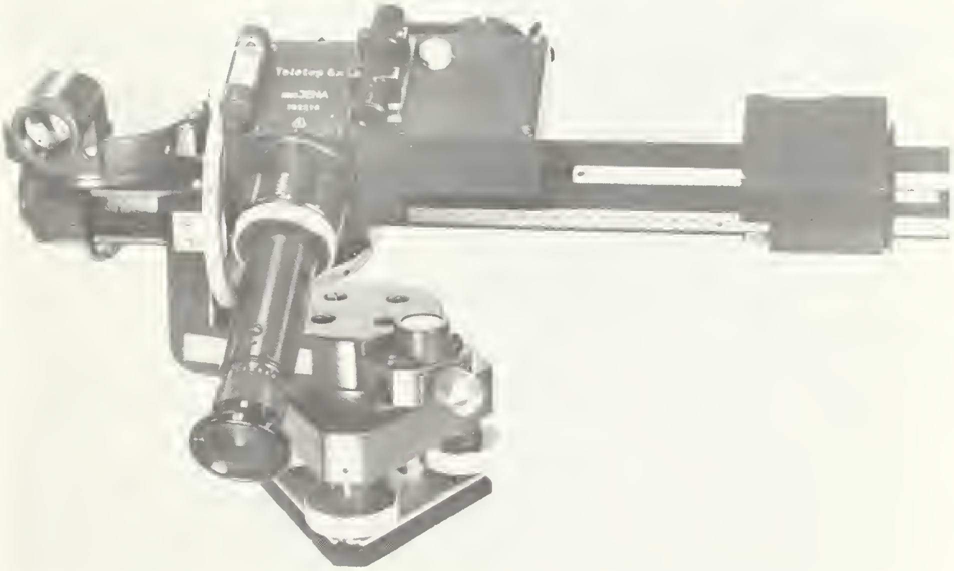


Figure 1.--The unmodified Zeiss Teletop.

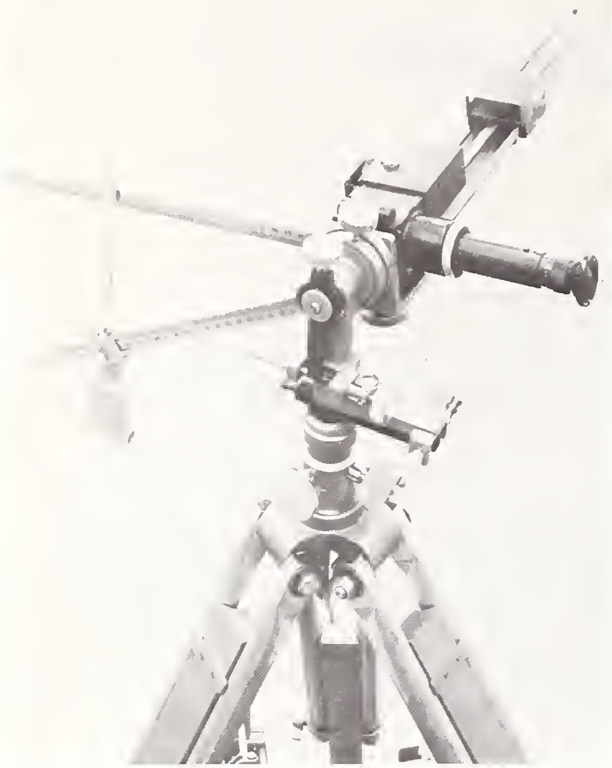


Figure 2.--Fricke's Universal Teledendrometer (UTD).

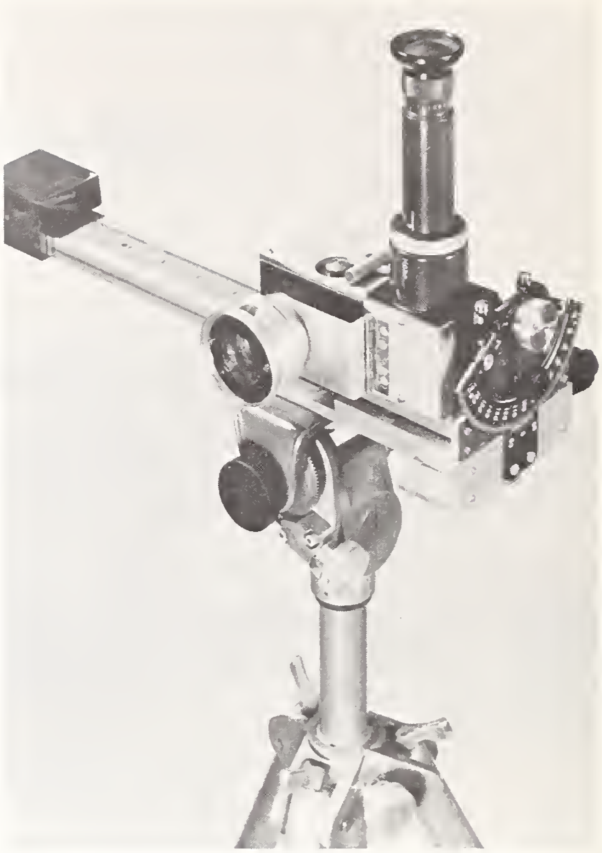


Figure 3.--Mesavage's modified Teletop dendrometer.

Teletop to develop the Universal Teledendrometer (UTD) (fig. 2). Mesavage's modified Teletop dendrometer (Mesavage 1969b) (fig. 3) incorporated improvements over both the unmodified Teletop dendrometer and Fricke's UTD.

Recently, the Todis dendrometer, manufactured in Germany by F. W. Breithaupt & Sohn, has been offered for sale in the United States. The Todis also uses a variable length rangefinder.

The Barr and Stroud Dendrometer

In 1947, Hummel and Laurie of the British Forestry Commission saw the possibility of using an optical rangefinder to measure tree diameters. Barr and Stroud Ltd. of Glasgow, Scotland, undertook development of a dendrometer based on a rangefinder originally used for tank gunnery. Hummel's (1951) paper mentioned trial of an early model of the Barr and Stroud dendrometer. In 1953 the Barr and Stroud dendrometer, type F.P. 7, was produced. It gave 5.5 image magnification, had a rangefinder base length of 8 inches, and an inclinometer graduated in sine of elevation angle. These features have been common to all subsequent models of the Barr and Stroud dendrometer. A major fault in the design of the F.P. 7 was the inability of the inclinometer to record an angle of depression.

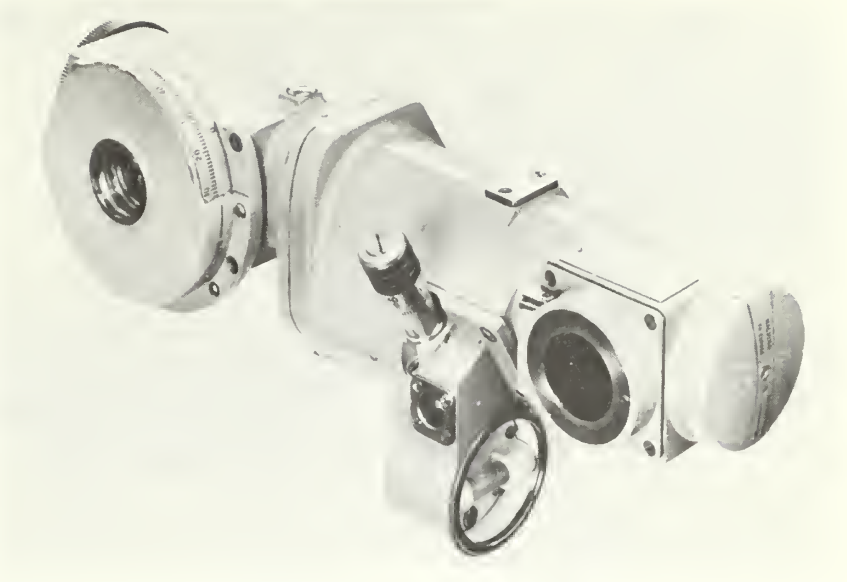


Figure 4.--The Barr and Stroud dendrometer, Model F.P. 9.

The F.P. 9 Barr and Stroud dendrometer (fig. 4) was the first model commercially available in the United States (Mesavage 1967). The F.P. 9 could measure angles of depression up to 25 degrees. The Barr and Stroud Model F.P. 12 dendrometer (fig. 5) improved on the F.P. 9 chiefly by being easier to use rather than giving more accurate measurement. The F.P. 15, most recent model of the Barr and Stroud dendrometer, incorporates refinements recommended by Hartman (1967) and Mesavage (1967). These include a translucent lens barrel for the scale-reading eyepiece to admit adequate

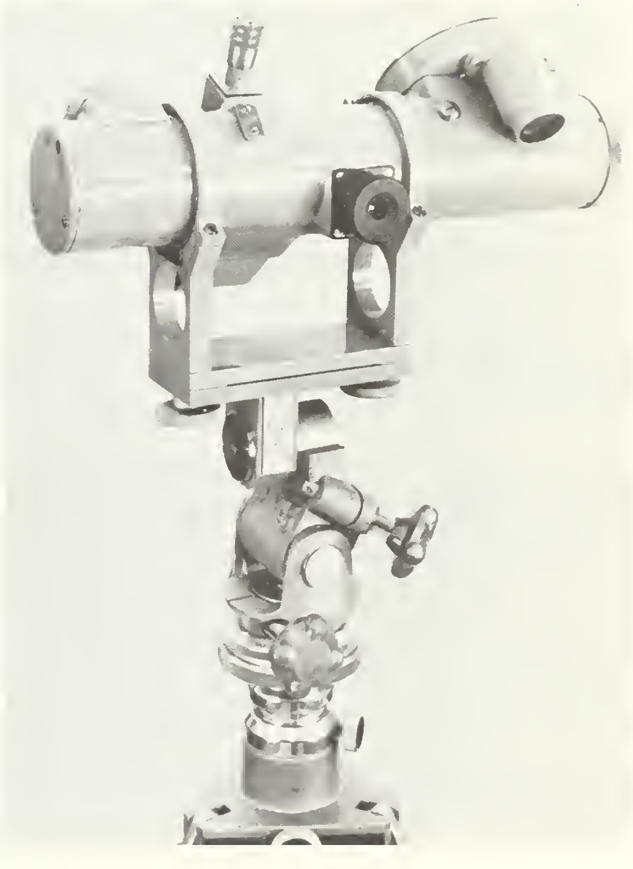


Figure 5.--The Barr and Stroud dendrometer, Model F.P. 12.

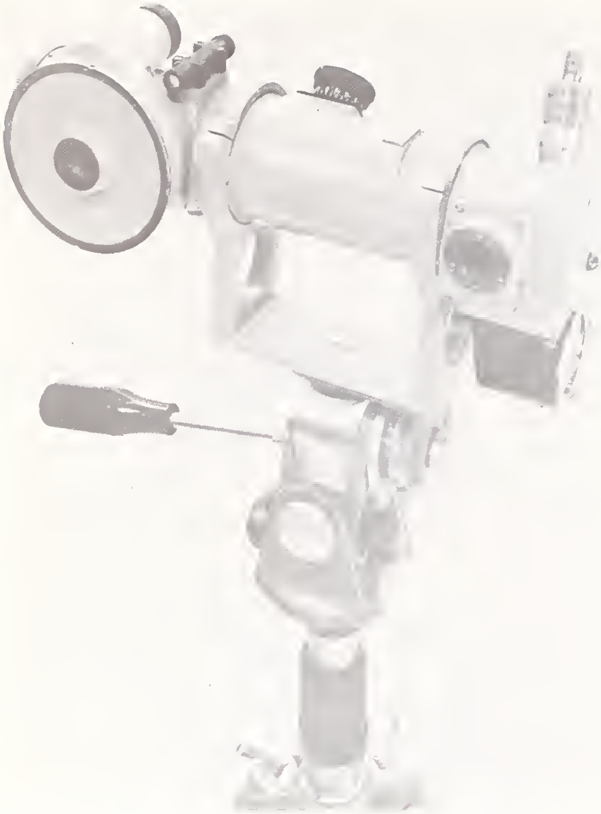


Figure 6.--The Barr and Stroud dendrometer, Model F.P. 15.

light in dense shade, and limits of 60 degrees for both elevation and depression angle. As shown in figure 6, a sight was added to the F.P. 15 for rapid aiming at the tree stem. Also, the inclinometer is graduated in terms of (1 + sin) to prevent confusion between negative and positive readings in the neighborhood of zero elevation (Mesavage 1969a).

The successive modifications that have led to a series of models of the Barr and Stroud dendrometer have not been basic changes of design; consequently, the latest instrument cannot be expected to be more accurate than its predecessors.

Various trials and use of the Barr and Stroud dendrometer have been reported by Jeffers (1956), Grosenbaugh (1963), Bouchon (1967), Bruce (1967b), Hartman (1967), Van Schie (1967), Arvanitis (1968), Bell and Groman (1968, 1971), Bower (1971), Langley and others (1971), Wensel (1971), and Wensel and Schoenheide (1971).

Presently existing literature on the use of the Barr and Stroud dendrometer shows a diversity of findings ranging from a serious deficiency in accuracy (Bouchon 1967) to excellent accuracy (Bell and Groman 1968, 1971). Some workers have obtained a satisfactory level of accuracy only by using the dendrometer in association with other instruments and increasing complexity of the measurement process to the point where efficiency of fieldwork is greatly reduced (Van Schie 1967). Others, while conceding the superior accuracy of the Barr and Stroud, consider its advantages over certain other instruments doubtful for practical work because of initial instrument cost and the rapidity with which measurements are made (Pfeiffer 1967; Arvanitis 1968).

In almost all cases cited in the literature where diameter measurements taken with the dendrometer differed from those taken with mechanical calipers, the dendrometer measurements were somewhat larger. This was the case even when the difference was of no statistical nor practical significance.

A TEST TO EVALUATE BIAS OF DENDROMETER DIAMETER MEASUREMENTS

To obtain a check on dendrometer measurements of diameter as precise as any that could be made under field conditions, an experiment was conducted in which three observers took dendrometer measurements and compared them with caliper measurements taken at the same point on each tree stem. The observers will be designated in this report by their respective initial, P, B, or R.

Nature of the Sample

Open stands of ponderosa pine (*Pinus ponderosa* Laws.) on level or nearly level terrain were chosen for the experiment to minimize extraneous difficulties. So little underbrush was present that it presented no obstacle to the use of the instrument. The experiment was conducted in clear weather and light conditions were excellent due to the open character of the stands. Several instrument locations were established in each of three stands. Sample trees were selected by systematically measuring d.b.h. of all normally formed trees visible within reasonable distance of the instrument location.

Measurements were taken of 88 trees, but one was subsequently rejected because of a dendrometry mistake. The trees ranged from 5.2 to 31.2 inches d.b.h. (caliper measurement) and were of normal form at the point of measurement.

Methods

A pair of wooden calipers, large enough to measure trees up to 36 inches in diameter, was carefully adjusted before being used in the experiment. Calipers were checked again after all tree measurements were taken and were still in proper adjustment. The dendrometer tested was an unmodified Barr and Stroud instrument, Model F.P. 12. Halving and coincidence adjustments were checked before and after measurement according to the manufacturer's instructions.

Each sample tree was marked at breast height before measurement. A strip of plastic flagging was carefully wrapped around the tree and tied so that the knot would not interfere with use of the calipers.

Care was taken with every tree to see that the diameter measured with the calipers was perpendicular to the line of sight from the instrument location to the tree and was, as nearly as possible, the diameter that would be measured with the dendrometer. Caliper measurements were agreed upon by all three observers.

After the caliper measurement of diameter was recorded, each observer took a set of dendrometer readings. After taking measurements, each observer disturbed the instrument setting so that the next man could establish his own coincidence settings. To assure independent measurements, no observer had prior knowledge of what instrument readings should be. This measurement routine was followed for each tree in the sample.

Compilation of Data

Tree diameters were computed from the corresponding dendrometer readings with the computer program NETVSL (Stage and others 1968), which uses formulas published by Grosenbaugh (1963) for his computer program, STX (Grosenbaugh 1964b, 1967b). NETVSL uses double precision arithmetic in evaluating the formulas. To perform such computation, a set of instrument constants is necessary. These constants serve as coefficients in the formulas. When the data for this study were compiled, three sets of instrument constants were available:

1. Those derived empirically and published by Grosenbaugh (1963) that assume a glass-refractive index of 1.523 and a mean prism-deflection angle for the instrument of 135.0 minutes;

2. Those derived later from the manufacturer's design parameters and published by Grosenbaugh (1967b) that use the glass-refractive index given by Barr and Stroud Ltd.; and

3. A set of constants computed by Grosenbaugh's (1963) exact method, using the exact glass-refractive index 1.5658 and a mean prism deflection angle (MPD) of 136.29 minutes. This MPD was obtained from direct calibration of the instrument used in this study.

The values of the constants in these three sets are shown in table 1. The instrument constants labeled B, Q, U, and G refer to characteristics of the instrument's design and manufacture. These constants are necessary to convert dendrometer readings to diameters and distances.

The data collected for this study were compiled three times, using the three sets of instrument constants; so diameters computed with each set of constants could be compared with the calipered diameters. Each of the three observers had three sets of 87 dendrometer diameters to compare with the caliper measurements. Caliper measurements were considered to be correct for the purpose of evaluating bias and error of dendrometry. So nine sets of 87 diameters were tested against the caliper measurements. Analysis of the data is described in appendix A.

Table 1.--*Values of the instrument constants in the three sets used for data compilation in comparing dendrometer diameters with calipered diameters*

Set of constants	Labeled instrument constants[1] B	Q	U	G
Grosenbaugh's old constants (GO)	8.0	0.01970203	-1.1933	1.523
Grosenbaugh's new constants (GN)	8.0	.01964673	-1.1905	1.5658
Direct calibration constants (DC)	8.0	.01982151	-1.2012	1.5658

[1] These constants, which refer to characteristics of the instrument's design and manufacture, are necessary to convert dendrometer readings to diameters and distances.

Discussion and Conclusions

The choice of instrument constants for the compilation of dendrometer data may not be critical, providing that the constants do not differ from the optimum set by more than the constants in this experiment differed from each other. In this experiment, three sets of instrument constants were used and, at one time or another, each seemed to provide satisfactory results. No significant differences were detected between the estimates obtained with each of the sets. Likewise, no significant difference was detected between the measurements made by three different observers.

Whatever its source, a positive bias of about 1.19 percent was found when dendrometer estimates of tree diameter were compared to caliper estimates of the same diameters. This bias would amount to about 2.39 percent bias in estimating a cross-sectional area of the stem. Grosenbaugh (1963) reported a basal area overestimate of about 0.7 percent in a similar trial involving fewer trees. Also, bias seemed to increase with the size of tree, hence its expression as a percentage above. The absence of a regression intercept different from zero indicates no constant component of bias that would remain the same for all trees.

The "pure" error remaining after bias is removed is also correlated with the diameter of the tree being measured. This was the reason for weighting the observations in regression calculations. An unqualified statement of error cannot be made. However, two-thirds of the time, the error discovered in this experiment was less than 0.03 inch for 5-inch trees and less than 0.2 inch for 30-inch trees. These results suggest that after any needed removal of bias, dendrometer diameter measurements taken by an observer working under good stand and weather conditions will be within a quarter of an inch of calipered measurements at least two-thirds of the time. This assumes the range of tree sizes usually encountered in the Mountain States.

Source of Bias

The source of the bias demonstrated by this experiment is not readily apparent. Possibly none of the instrument constants used were quite applicable. Certainly, they could be adjusted, perhaps to some least squares criterion of agreement between dendrometer and calipers as Grosenbaugh (1963) suggested. However, the fact that the three sets of instrument constants shown in table 1 gave essentially the same results suggests that fairly severe adjustment might be necessary to obtain unbiased measurements. It is questionable whether such methods would be justified to adjust parameters that have been derived exactly from known standards of manufacture. Apparently, no dissatisfaction with the two previously published sets of constants has been registered by those who have used the Barr and Stroud dendrometer. The possibility of adjusting the instrument constants in no way establishes them as the source of bias discovered in this experiment.

Another possible source of bias is the difference between the tree diameter measured with the calipers and the two radii estimated with the dendrometer. It is assumed that the angle measured with the dendrometer exactly subtends a circle (the tree cross section) with two radii perpendicular to the points at which the sides of the angle are tangent to the circle.

Most tree cross sections are more nearly oval than circular (Müller 1957; Chacko 1961); nevertheless, when a diameter is assigned to a tree stem, a circular shape is necessarily implied. If one number must be used to describe this "diameter," then it is best if the number, D, is such that $\pi/4\ D^2$ gives the cross-sectional area of the tree even though it is not circular. If the cross section is elliptical, such a diameter can be computed by

$$D = 2\sqrt{ab} ,$$

where a and b are the semimajor and semiminor axes of the ellipse (Goosens 1967). This pseudodiameter adequately serves the purposes of tree volume calculation because it gives the true area of the appropriate ellipse when used in the formula for area of a circle. Apparently, dendrometer diameter estimates of noncircular stems are biased. However, Grosenbaugh (1963) stated that the magnitude of the bias is only slightly more serious than similar biases that foresters have long tolerated with mechanical forks, calipers, or tapes. For elliptical trees having the short diameter nine-tenths of the long diameter, an overestimate in cross-sectional area of about 0.4 percent will be obtained with a diameter tape (Chaturvedi [1926]; the Imperial Forestry Institute [1944]). Apparently, the bias in dendrometer measurements caused by noncircular stems is not much larger than that.

However, this explanation of the bias discovered by this experiment is not supported by the nature of the bias indicated by the regression equation. The baseline of the dendrometer's rangefinder is 8.0 inches; so when the diameter of the subject tree is 8.0 inches, no angle exists. The lines of sight from the dendrometer are parallel; so the instrument becomes, in effect, an optical caliper. Only when measuring trees of this size are the two radii measured with the dendrometer coincident with tree diameter perpendicular to the line from the center of the tree to the instrument location--the diameter measured with calipers in this experiment. For this reason, the expected apparent bias would be zero for trees 8.0 inches d.b.h. One would then expect a negative intercept in the regression of dendrometer diameter on calipered diameter so that the regression would pass through the point ($D_c = 8.0$, $D_d = 8.0$) and the weighted grand mean. Therefore, an underestimate of diameters by the dendrometer would be implied for trees smaller than 8.0 inches d.b.h. The fact that trees approximately 8.0 inches by caliper measurement also receive an overestimate casts doubt on this explanation of any substantial component of bias.

Rounding or truncation error in the calculation of diameters from dendrometer readings might attach a bias to the resulting diameters. However, calculations for this experiment were done with double precision arithmetic on an IBM 360 computer (Stage and others 1968). At least 15 significant digits were guaranteed in computation; so it is unlikely that rounding error could have any noticeable effect on computed diameters. Although they were not used for comparisons, dendrometer diameters were also read from tables furnished by Barr and Stroud Ltd. Tabular diameters are less accurate than those computed with the formulas but, nevertheless, they showed a tendency for dendrometer diameters to exceed those taken with mechanical calipers.

Such overestimates could also result from inaccuracies inherent in human vision. Stage (1962) conducted experiments on the displacement of a tree stem image by the wedge prism which showed the presence of personal bias. Joseph P. McClure (personal communication) described an experiment in which observers were asked to move a prism back and forth along a line of sight until they judged from image displacement that the distance to a particular tree made it a marginal tree. Measurement of the distance with a tape usually showed that the distance was actually greater than the limiting distance computed from the tree's diameter. Ross (1968) took diameter measurements on some test trees with a Wheeler penta prism caliper in two ways, and found a statistically significant difference between diameters measured by the two methods. The instrument and the principles of its operation are exactly the same in both cases; so I attribute the difference between average measurements to some human characteristic.

Jeffers (1956) reported that some observers slightly overestimated tree diameters with the Barr and Stroud Model F.P. 7 dendrometer. Van Schie (1967) reported similar overestimates with a later model, as did persons using dendrometers based on the Zeiss Teletop (Fricke 1967; Bower 1971).

Such bias might be caused by poor definition of a tree's edge resulting from rough bark, but use of plastic flagging to provide a clearly defined edge did not eliminate the bias. In some cases, mechanical calipers could compress the bark, thereby providing check measurements somewhat smaller than those taken with the dendrometer. But, on trees with tight, hard bark, considerable pressure would be required to close steel calipers to 0.1 inch less than the correct measurement. Bark probably could not be compressed without bending wooden calipers.

It may well be that the bias in dendrometer measurements shown by this experiment arises from several sources. The information obtained from this experiment permits no definitive statement about dendrometer bias beyond the fact that it does exist, is of given magnitude, and is essentially the same for three different observers.

DENDROMETER MEASUREMENTS OF DIAMETER, HEIGHT, AND VOLUME UNDER AVERAGE FIELD CONDITIONS

The Study Area

Average field conditions in northwestern Montana are not the most pleasant for fieldwork. Steep hills, a dense understory in many places, and much fallen timber result in challenging conditions for testing the accuracy of an instrument. Some trees, especially Engelmann spruce (*Picea engelmannii* Parry) and the firs (*Abies* spp.), have dense crowns that severely limit visibility of upper stems. Yet, these are average field conditions for the area, and it was under such conditions that the trial described was carried out. Tests conducted in an open stand of pines might minimize observer error, but they would hardly test the feasibility of the instrument's use under average conditions, unless open pine stands constitute the average condition.

Procedures

Sample Tree Selection

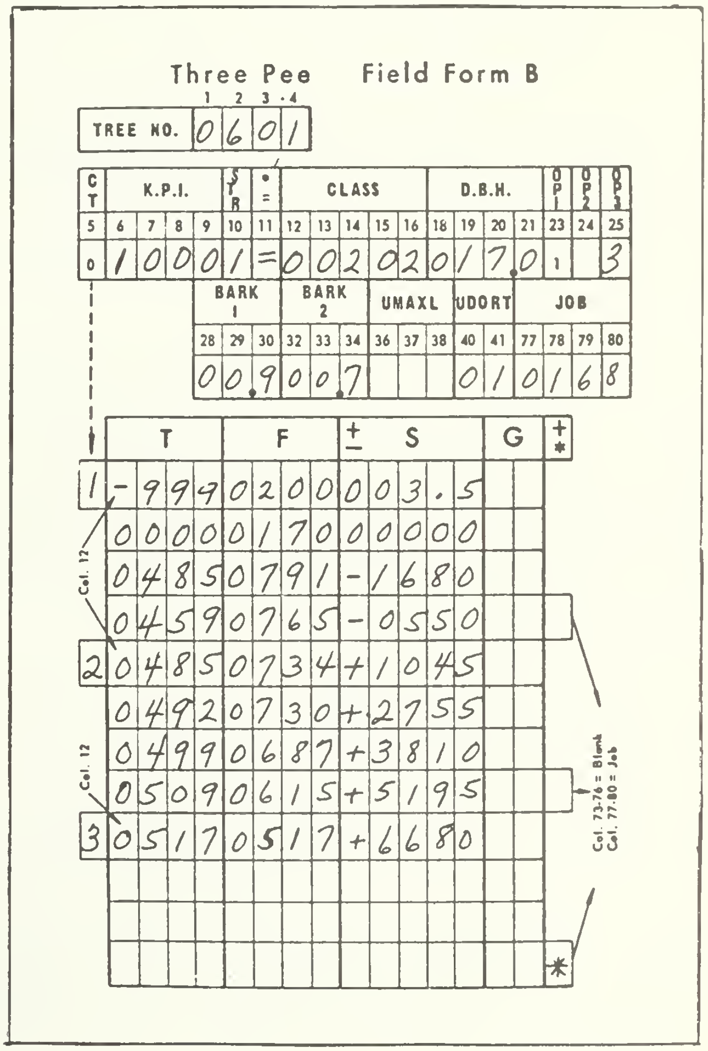
Field tests of the Barr and Stroud dendrometer were part of a study designed to test net volume sampling methods. Methods involved felling some trees at each location to estimate cull volume and measuring others with the dendrometer to estimate gross volume. By first measuring some trees to be felled with the dendrometer, we were able to examine the precision and accuracy of dendrometer measurements, and to decide what adjustment, if any, might be required.

At that time, the sampling unit for forest inventories used by the Forest Service's Northern Region was a cluster of 10 variable radius plot sampling points, often referred to as a "location." Each sampling point within the cluster was 70 feet from the nearest adjacent point. Trees were selected for measurement from each point with a critical angle corresponding to a basal area factor of 40. From the trees tallied for basal area estimation on each of 39 locations, a subsample was selected to be felled and measured for purposes of cull volume estimation. Another subsample was selected for measurement with the dendrometer to provide a gross volume estimate of the trees that were not felled. It was further specified that of the trees to be felled on each location, those with the largest and smallest diameter at breast height would be measured with the dendrometer. This was a means of checking dendrometer measurements. Trees of abnormal form or having defective boles were not purposely excluded. Thus, a sample was obtained of 78 trees, which were measured with the dendrometer while standing and after felling. Diameter breast height ranged from 5.1 to 43.1 inches.

Dendrometer Measurement Procedures

Dendrometer measurements were taken in the following manner. Stump diameter outside bark was measured with a diameter tape at a point 1 foot above the ground. Then, a strip of plastic flagging was wrapped around the trunk at breast height (4.5 feet) and d.b.h. was measured with a diameter tape. Two measurements of bark thickness at breast height were taken at right angles to each other with a Swedish bark punch. The sum of these two bark thickness measurements was the estimate of double bark thickness at breast height.

The dendrometer then was set up at a suitable instrument location. Diameter at breast height was measured with the dendrometer. Flagging not only marked breast height but provided a clearly defined target for taking true and false coincidence readings. Measurements at other heights on the stem were then taken at certain points above breast height. Because the tree crown sometimes hides portions of the bole, it is not practical to prescribe fixed heights at which dendrometer measurements will be taken. However, wherever possible, measurements were taken at no fewer than eight levels on the tree stem and at the top of the tree to accurately establish total height. Most trees had five measurements taken between breast height and the tree's top. Dendrometer measurements were recorded on a field form designed by Floyd Johnson of the Pacific Northwest Forest and Range Experiment Station (fig. 7).



Three Pee Field Form B

	1	2	3	·4
TREE NO.	0	6	0	1

C T	K.P.I.				S T R	• =	CLASS					D.B.H.				O P 1	O P 2	O P 3
5	6	7	8	9	10	11	12	13	14	15	16	18	19	20	21	23	24	25
0	1	0	0	0	1	=	0	0	2	0	2	0	1	7	0	1		3

BARK 1			BARK 2			UMAXL			UDORT		JOB			
28	29	30	32	33	34	36	37	38	40	41	77	78	79	80
0	0	9	0	0	7				0	1	0	1	6	8

	T				F				± S					G		+ *
1	-	9	9	9	0	2	0	0	0	0	3	.	5			
	0	0	0	0	0	1	7	0	0	0	0	0	0			
	0	4	8	5	0	7	9	1	-	1	6	8	0			
	0	4	5	9	0	7	6	5	-	0	5	5	0			
2	0	4	8	5	0	7	3	4	+	1	0	4	5			
	0	4	9	2	0	7	3	0	+	2	7	5	5			
	0	4	9	9	0	6	8	7	+	3	8	1	0			
	0	5	0	9	0	6	1	5	+	5	1	9	5			
3	0	5	1	7	0	5	1	7	+	6	6	8	0			
																*

Col. 12

Col. 12

Col. 73-76 = Blank
Col. 77-80 = Job

Figure 7.--The form used to record dendrometer measurements in the field.

Measurement of Felled Trees

Diameter and bark thickness measurements on felled trees were taken at stump height (1.0 foot), breast height (4.5 feet), and at alternating 8.1- or 8.2-foot intervals on the stem. This spacing was used to obtain measurements at intervals of 16.3 feet because National Forest personnel who assisted in the collection of data wished to apply their log scaling rules to felled trees. In some cases, the stem was cut and measured at 4-foot intervals to determine the location of cull volume. Diameter measurements were taken across the cut section of the stem on two axes that best represented the cross-sectional area of the stem. In the absence of deformities or abnormalities on the stem, these were the long and short axes of the cross section. Both diameter inside bark and diameter outside bark were measured by stretching a steel tape across the cut section of the tree trunk. The cross-sectional area of felled tree stems was computed as though the cross section were elliptical in shape. That is,

$$A = \pi ab$$

where

A = cross-sectional stem area (either inside or outside bark, depending upon whether or not a and b include bark)

a = the semimajor axis of the assumed ellipse

b = the semiminor axis of the ellipse

The cross section of a tree stem can more generally be described as an oval, not necessarily elliptical or circular. Matérn (1956) generated six such ovals (including an ellipse) by functions in polar coordinates. Their exact area was calculated and compared with area estimates obtained from 10 different methods of tree diameter measurement. For five of the six ovals, the diameter obtained from the geometric mean of measurements on the maximum and minimum axes gave the smallest bias in estimation of area. The diameter used in this study to describe the tree trunk at various points was:

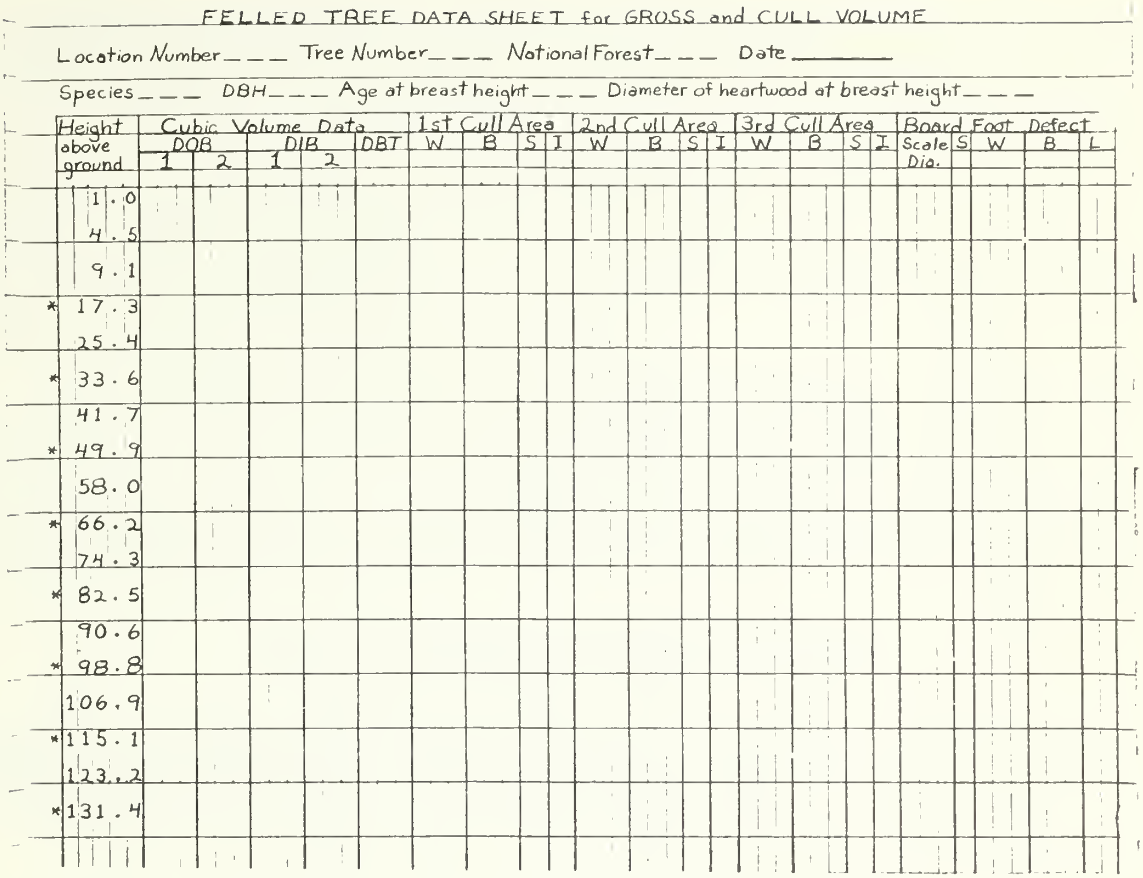
$$D_f = 2\sqrt{ab}$$

When used in the formula for area of a circle, this diameter gives the tree's cross-sectional area as nearly as is practical with no more than two measurements. Only measurements on the long and short axes of the cross section were recorded in the field. Area and the corresponding pseudodiameter were computed during data compilation. Data were recorded on a specially prepared form (fig. 8). Felled tree measurements were recorded in feet and in tenths and hundredths of feet.

Tree Volume Computation

A computer program was used to obtain volumes for both felled trees and those measured with the dendrometer. For a description of the program and its use, the reader is referred to Stage and others (1968). The computer program NETVSL (net volume, surface, and length) can be used to compute gross, cull, and net volumes, as well as surface area of trees from measurements of stem length and diameter. Measurements on standing trees cannot be used to compute cull volume.

By proper selection of its built-in options, the program can be used to compute total cubic volume in the tree, cubic volume between a specified stump height and a specified top diameter, and board-foot volume between a specified stump height and top diameter. Board-foot volume can be computed according to the Scribner rule, the International 1/4-inch rule, or both. Various combinations of differently specified



FELLED TREE DATA SHEET for GROSS and CULL VOLUME

Location Number___ Tree Number___ National Forest___ Date______

Species___ DBH___ Age at breast height___ Diameter of heartwood at breast height___

Height above ground	Cubic Volume Data					1st Cull Area				2nd Cull Area				3rd Cull Area				Board Foot Defect				
	DOB		DIB		DBT	W	B	S	I	W	B	S	I	W	B	S	I	Scale Dia.	S	W	B	L
	1	2	1	2																		
1.0																						
4.5																						
9.1																						
* 17.3																						
25.4																						
* 33.6																						
41.7																						
* 49.9																						
58.0																						
* 66.2																						
74.3																						
* 82.5																						
90.6																						
* 98.8																						
106.9																						
* 115.1																						
123.2																						
* 131.4																						

Figure 8.--The form used to record felled-tree measurements in the field.

volumes can be computed in one execution of the program. For this study, all comparisons of tree volume estimates were based upon cubic volume of the total stem, including bark. Inaccurate estimates of bark thickness can bias the estimate of tree volume inside bark, yet this bias has nothing to do with dendrometer accuracy.

For computation of total tree volume, the volume below the first diameter measurement is computed as though the stump were a cylinder of the given length and diameter. The volume of the section terminating at the tip of the tree is computed as though the section were a right cone. Intervening sections of normal taper are treated as right parabolic frustrums, and their volume in cubic feet is computed by Smalian's formula. If the diameter at the top of a stem section is larger than the next lower diameter, indicating negative taper, the intervening section is treated as an inverted right conic frustrum. This geometric model has serious shortcomings, especially for calculating volume in the lower portion of a tree. Further, if these deficiencies are ignored, the results of this study could be misinterpreted.

Dendrometer D.B.H. Versus Taped D.B.H. Under Average Field Conditions

On each of 403 trees measured with the dendrometer (including the 78 trees subsequently felled), two measurements were taken of d.b.h.--one with the dendrometer and one with a steel diameter tape. Breast height was marked by a piece of plastic flagging, measured with a tape, and recorded. Then, the dendrometer was set up at a suitable location and d.b.h. was measured and recorded again. The linear regression of d.b.h., measured with the dendrometer on taped d.b.h., was fitted to the sample of paired observations. To attain homogeneity of variance around the regression line, it was necessary to assign each pair of observations a weight inversely proportional to the square of the taped measurement. Weights were scaled so that the sum of weights was equal to the number of observations. The fitted regression was:

$$\hat{DBH}_d = \hat{b}_0 + \hat{b}_1 \; DBH_t$$

where

$\hat{b}_0 = 0.10712$

$\hat{b}_1 = 0.996676$

S_a = standard error of $b_0 = 0.0492664$

S_b = standard error of $b_1 = 0.0050488$

r^2 = coefficient of determination = 0.989815

For testing the differences of $\hat{b}_0$ and $\hat{b}_1$ from zero and unity, respectively, the t statistics are:

$$t_{b_0} = \hat{b}_0/S_a = 2.1743$$

$$t_{b_1} = (1 - \hat{b}_1)/S_b = 0.658275$$

The hypothesis that the regression intercept is not different from zero is rejected, but the hypothesis that the slope of the regression line is not different from unity is retained. If $b_1 = 1.0$, then the estimate of the regression intercept is the difference between means of the variables. The weighted means are:

$$\overline{DBH}_t = 8.8289899 \text{ inches}$$

$$\overline{DBH}_d = 8.9017811 \text{ inches}$$

giving an estimated regression intercept of 0.07779 inch. This figure indicates a positive bias in dendrometer diameter estimates that is less than 0.1 inch and is not correlated with tree size. As a percentage of diameter, this bias would decrease from 1 percent for trees of 7.8 inches d.b.h. to about 0.25 percent for trees of 30 inches d.b.h. For most work, this bias is not large enough to cause concern.

When dendrometer measurements were compared with caliper measurements, positive bias was a constant percentage of diameter. The reason for correlation between diameter and bias in one case and not in another is not evident. Even on trees of irregular cross-section shape, Matérn (1956) has shown that the expected value of a single caliper measurement is the same as the measurement obtained with a diameter tape. The caliper measurement would carry a larger error, but the mean diameter of a sample of trees would be expected to be the same for both measurement methods; so the standard against which the dendrometer measurements of diameter were compared had the same expected value in both cases (Brickell 1970a).

Barr and Stroud give the "approximate uncertainty of observation" for diameters measured with the dendrometer as 0.1 inch for trees up to 10 inches in diameter and 1 percent for diameters larger than 10 inches. They do not explain why it is constant up to a particular size and proportional to diameter beyond that size.

The arithmetic mean diameter of the 87 calipered trees was 17.7 inches and the unweighted arithmetic mean of the 403 taped trees measured on inventory locations was

13.7 inches. In one sample, trees may tend toward sizes for which the uncertainty of observation is a constant 0.1 inch, whereas in the other sample, larger trees may tend toward a percentage uncertainty. Weights given the observations for regression in both samples would have given small trees more influence on the regression line than large trees. Of the 87 calipered trees, only seven trees, about 8 percent, were 10 inches or less in diameter; but 154, about 38 percent of the taped trees, were 10 inches or less in diameter. Unfortunately, this explanation of the nature of diameter estimation bias will not suffice if the term "uncertainty of observation" as used by Barr and Stroud means random error. Unless error and bias of dendrometer measurements are related, this study cannot explain the apparently different forms of diameter measurement bias in the two samples. However, regardless of the form of the bias, it is positive (an average overestimate) in both cases.

Plastic flagging wrapped about the tree at breast height was intended to allow more accurate setting of true and false coincidence. It may have increased accuracy and precision, but reduction of the uncertainty in coincidence settings due to a rough and indistinct bark edge did not remove all sources of bias. However, flagging is recommended in measuring trees with the dendrometer as it serves a practical purpose: the tree to be measured can be easily seen against a forest background. If adjacent trees are of the same species and size class, it is sometimes easy to become confused if some sort of marker is not used.

Bias and Precision in Height Estimation

Height measurements for 6 of the 78 trees measured with the dendrometer and subsequently felled were obviously mistakes. These were set aside. Remaining height measurements yielded the following statistics:

$\overline{H}_d = 78.118042$ feet $\qquad S_{H_d} = 27.692017$ feet

$\overline{H}_f = 77.740265$ feet $\qquad S_{H_f} = 27.782715$ feet

where

$\overline{H}_d$ = mean of dendrometer height measurements

$\overline{H}_f$ = mean of taped height measurements after felling

S_{H_d} = standard deviation of dendrometer measurements

S_{H_f} = standard deviation of taped measurements.

The mean difference $(\overline{H}_d - \overline{H}_f)$ equals 0.377777 foot. The correlation coefficient of the two sets of measurements is 0.99821. So the standard deviation of differences between taped and dendrometered measurements is:

$$S_{(H_d - H_f)} = \sqrt{S^2_{H_d} + S^2_{H_f} - 2\,(0.99821\; S_{H_d} S_{H_f})}$$

$$= 1.6622 \text{ feet.}$$

The standard error of the mean difference obtained by division:

$$S_{\overline{(H_d - H_f)}} = \frac{S_{(H_d - H_f)}}{\sqrt{72}} = 0.19588 \text{ foot.}$$

To test the hypothesis that no constant bias exists in height measurements with the dendrometer, Student's t test was applied. The t statistic is obtained by division:

$$t = (H_d - H_f) \;/\; S_{\overline{(H_d - H_f)}}$$

$$= 1.9286.$$

Thus calculated, t is less than the value necessary to reject the hypothesis at the 95 percent confidence level; so the hypothesis of no bias was retained.

To illustrate further the absence of bias in dendrometer height measurements, the regression

$$\hat{H}_d = \hat{b}_0 + \hat{b}_1 H_f$$

was fitted to the paired measurements with the result:

$\hat{b}_0 = 0.77003$	$S_{b_0} = 0.58722021$
$\hat{b}_1 = 0.99495429$	$S_{b_1} = 0.00711863.$

The t statistics

$$t_0 = \hat{b}_0 / S_{b_0} = 1.3113138$$

$$t_1 = (1 - \hat{b}_1) \;/\; S_{b_1} = 0.708803$$

do not justify rejection of the hypothesis that the coefficients b_0 and b_1 equal zero and unity, respectively, and that, therefore, H_d equals H_f.

If $b_1 = 1$, then there is no correlation between tree height and measurement differences--between $(H_d - H_f)$ and H_f. Also, variance about the regression was not related to tree size.

This finding that the Barr and Stroud dendrometer provided an unbiased means of estimating tree heights differs from results of Bouchon's (1967) study. In that case, heights measured with the dendrometer were compared with heights of the same trees measured with a Wilde-TO theodolite. Sample trees ranged from 5 to 98 meters in height. An important negative bias was observed, the mean of which was -80.5 centimeters for 20 trees.

Bouchon found that this bias was caused by a consistent underestimate of the horizontal distance from the instrument location to the subject tree. Conventional practice uses slope distance from the instrument to the point of measurement with the sine of angle of elevation to compute tree height. When a chained distance was substituted for the one obtained from the dendrometer measurements, the bias virtually disappeared. Before such correction, the standard deviation of differences in height measurement was 58.4 centimeters (1.92 feet); after correction, it was 3.26 centimeters. Thus, while the standard deviations of the differences (measured height — actual height) do not seem to differ greatly between the present study and Bouchon's, the French experiment disclosed a bias in height (or distance) measurement not encountered in this study. The reason for Bouchon's underestimate of horizontal distance is not apparent.

Jeffers' (1956) trial of the Model F.P. 7 Barr and Stroud dendrometer disclosed no bias in estimates of distance or height. For differences between true measurements and dendrometer measurements taken by several observers on a number of trees, Jeffers gives approximate standard deviations of 1 foot in range and one-half foot in height. It is assumed that these are standard deviations about the true measurements. Barr and Stroud (*Pamphlet 1217K*) give the approximate uncertainty of observation under average practical conditions as 1.5 percent for height.

In this study, the standard deviation of differences between dendrometer height and true height, expressed as a percentage of mean true height, is:

$$(S/\bar{H}_f) \times 100 = (1.6622/77.74) \times 100 = 2.14 \text{ percent.}$$

This exceeds the Barr and Stroud figure, but not by an important amount.

It may be worthwhile to compare test statistics obtained for the dendrometer in this study with the results of tests conducted on other height measuring instruments. Warren (1958) examined accuracy and precision of the Abney level, the Haga altimeter, and the Blume-Leiss altimeter (Parde 1955). Five observers measured the same 15 trees in each of three different stands. The trees were climbed and measured with a tape to provide standard heights with which the instrument measurements could be compared. Hunt (1959) used a transit to establish standard heights of a number of trees. Measurements were then taken with the Haga altimeter, the Engineers hypsometer (similar to the Forest Service hypsometer [Krauch 1918], but smaller), the Abney level, and the Spiegelrelaskop (Daniel and Sutter 1955; Bitterlich 1958). The mean differences between actual height and instrument height and standard deviations of the differences for the instruments examined by Hunt and Warren are shown in table 2. Those mean differences and standard deviations obtained by the Barr and Stroud dendrometer are

Table 2.--*A summary of results obtained from several tests of tree height measuring instruments*

Instrument	Number of measurements	Mean difference[1] (instrument-actual)	Standard deviation of differences
		Feet	*Feet*
Abney level (Warren)	225	0.369	1.76
Haga altimeter (Warren)	225	-.144	1.91
Blume-Leiss altimeter (Warren)	225	-.319	1.93
Haga altimeter (Hunt)	351	.29	1.22
Engineer's hypsometer (Hunt)	351	-.11	1.60
Abney level (Hunt)	351	-.26	1.62
Spiegelrelaskop (Hunt)	351	.38	1.51
Barr and Stroud dendrometer	72	.378	1.66

[1] Actual height is for Warren's experiment, a taped measurement obtained by climbing; for Hunt's experiment, a height measured with a transit; and for the Barr and Stroud dendrometer, a taped measurement after felling the tree.

shown for comparison. Table 2 does not show statistical significance of these differences, but practical significance can be judged from the magnitude of the figures. All mean differences are less than one-half foot in absolute value and all standard deviations of differences are between 1 foot and 2 feet. Furthermore, so far as the dendrometer measurements are concerned, no correlation was found between measurement error and tree height. In other words, tall trees can be measured as accurately as short trees.

If it is not correlated with height, an average measurement error of less than one-half foot in tree height will introduce little bias into tree volume estimates.

Fitje (1967) reported tests of several height measuring instruments in Norway, the Blume-Leiss altimeter, the Suunto clinometer, and an instrument called the Meridian, which are capable of about the same degree of accuracy. Some observers detected no bias in height measurement. In other cases, a bias as large as 0.35 meter was discovered. The standard deviations of differences between instrument height and actual height ranged from 0.4 to 1 meter. Neither bias nor error was found to be correlated with tree height. In some instances, Fitje's experiment shows somewhat less accuracy for the instruments tested than do tests conducted by Hunt and Warren. However, the most accurate observers obtained results comparable to those in table 2.

There seems to be little difference in accuracy and precision between the Barr and Stroud dendrometer and some of the other instruments generally used for measuring tree height. However, table 2 can be misleading. The comparison shown is between the dendrometer in use in generally steep and brushy terrain and other instruments used under carefully controlled conditions in selected stands on level or nearly level ground. Much of the error in height measurement with instruments like the Abney level is the result of inaccuracy in measuring horizontal distance or slope distance from the observer to the subject tree. On level terrain, as in Hunt's and Warren's tests, distance measurement may not be too difficult, but on steep or broken ground where heavy underbrush makes clearing a 100-foot path to every tree expensive, an error of several percent in distance measurement is not surprising. Slope distance to the tree top calculated from dendrometer readings may be more accurate under such circumstances.

If measurements of tree height taken with the Barr and Stroud dendrometer are unbiased, then bias observed in dendrometer volume estimates must arise from some other source.

Tree Volumes Based on Dendrometer Measurements

Expected Effect of Bias and Error in Diameter Measurement

The proportional bias in diameter measurement, 1.19 percent more than the corresponding calipers measurement, is greater for most trees than the constant bias of 0.078 inch discovered in comparisons of taped d.b.h. measurements. To avoid underestimating the effect of diameter measurement bias on calculated volume, the larger bias is used. To assess the effect of bias, it can be assumed that Huber's formula is a satisfactory model for tree stem sections. Then, cubic volume in the ith section of the bole is:

$$v_i = k\, d^2_i l_i$$

where

v_i = cubic volume in the ith section

d_i = midpoint diameter of the ith section

l_i = length of the ith section

k = $(\pi/4)/144$.

Now, if all stem sections have the same length, total cubic volume in the tree is given by:

$$\sum v_i = k\ l \sum d_i^2$$

If the diameter measurements used are actually 1.19 percent larger than they should be and section length is unbiased

$$\begin{aligned}\sum v_i' &= kl \sum (1.0119 d_i)^2 \\ &= kl\ (1.0239) \sum d_i^2\end{aligned}$$

and

$$\sum v_i' / \sum v_i = 1.0239$$

indicating an expected bias in volume measurements of 2.39 percent.

In theory, cross-sectional stem area, hence volume, will also have a component of bias resulting from the random error associated with diameter measurements.

If

$$D_d = 1.0119\ D_a + \varepsilon$$

where

D_d = dendrometer diameter

D_a = actual diameter

ε = a random error with $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \delta^2 D_a^2$ (Variance about regression is proportional to D_a^2)

then

$$\begin{aligned}D_d^2 &= (1.0119\ D_a + \varepsilon)^2 \\ &= 1.0239\ D_a^2 + 2.0238\ D_a \cdot \varepsilon + \varepsilon^2\end{aligned}$$

and

$$E_\varepsilon(D_d^2) = 1.0239\ D_a^2 + 2.2038\ D_a \cdot E_\varepsilon(\varepsilon) + E_\varepsilon(\varepsilon^2)$$

where E_ε denotes expectation with respect to the random variable ε. Since the expected value of ε is zero,

$$E_\varepsilon(D_d^2) = 1.0239\ D_a^2 + \delta^2 D_a^2$$

with $\delta^2 D_a^2$ being the additional component of bias. As given previously, estimated δD_a is about 0.2 inch for a 30-inch tree; or

$$0.2 = \delta \cdot 30 \rightarrow \delta D_a = 0.00667\ D_a$$

so that

$$\delta^2 D_a^2 = 0.000044\ D_a^2.$$

Though theoretically present, this component of bias is trivial and can be disregarded.

Effect of Bias in Height Measurement on Volume Measurement

Tree volume is nearly proportional to height. Thus, a 2 percent bias in height measurement, for example, would be carried through as 2 percent bias in volume measurement if it were evenly distributed throughout the tree stem. A constant height bias would lead to a larger volume bias for larger trees, but a smaller percentage of total volume, because it would be multiplied by a larger average cross-sectional stem area.

Where no bias is found in dendrometer height measurements, it can be inferred that there will be no nonrandom influence on volume measurement.

Comparison of Dendrometered Volumes (Outside Bark) With Volumes Measured After Felling

For 2 of the 78 trees measured with the dendrometer and subsequently felled, large differences between the two kinds of volume measurement obviously resulted from mistakes. The remaining 76 trees ranged in volume (felled tree measurement) from 2.6 to 380 cubic feet. Four dendrometer height measurements were not as accurate as the others. However, mistakes in height measurement apparently occurred in the topmost sections where they had little effect upon total tree volume; so the trees were allowed to remain in the sample because the value of the information contributed exceeded any possible distortion of results.

A preliminary regression of dendrometered volumes on felled tree volumes showed that variance about the regression line was proportional to the square of the independent variable, actual volume. Also, the estimated regression intercept was not statistically different from zero. A linear relationship passing through the origin of coordinates can be expected from this kind of data. The properly weighted regression conditioned to pass through the origin was:

$$\hat{V}_d = 1.05425\ V_f$$

where

$\hat{V}_d$ = estimated tree volume measured with the dendrometer

V_f = tree volume measured after felling.

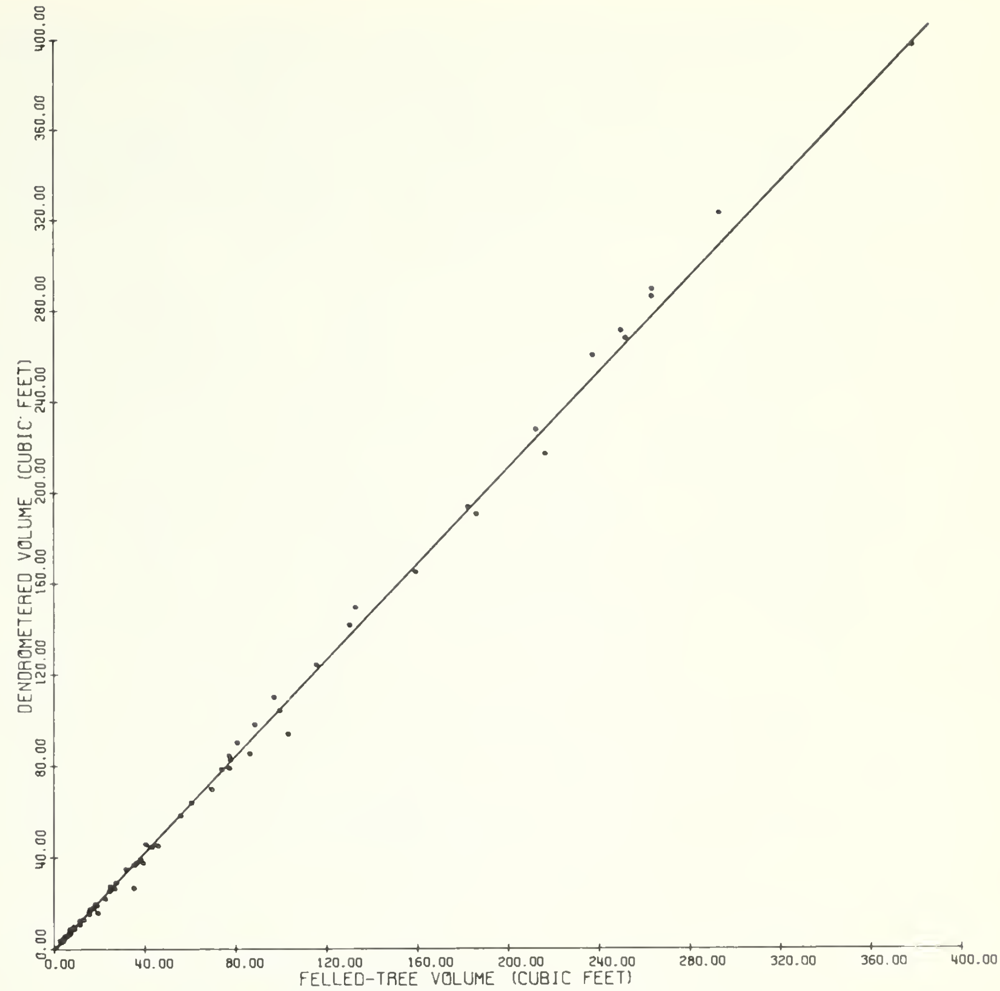


Figure 9.--Dendrometered volume plotted over felled-tree volume, with the fitted regression line.

This relationship is shown in figure 9. The estimated standard error of the regression coefficient (S_b) was 0.0088. The sample coefficient of correlation between the two kinds of volume measurements was 0.9974. The weighted root mean square residual about the regression was 0.606 cubic foot. However, this is a rather meaningless statistic when variance is not homogeneous. The standard deviation of residuals for various tree sizes as computed from the relationship between variance and tree volume is:

Tree volume	*Standard deviation of residuals*
Cubic feet	
10.0	0.30
50.0	1.52
100.0	3.04
150.0	4.55
200.0	6.07
250.0	7.58
300.0	9.11
350.0	10.62
400.0	12.14

Standard error is about 3 percent of tree volume, which agrees substantially with results of Grosenbaugh (1963) and Arvanitis (1968).

The regression coefficient of 1.05425 indicates an average positive bias in dendrometer volume measurements of 5.4 percent Even if twice its standard error is subtracted from the coefficient to obtain the coefficient's lower probable bound, a bias of 3.7 percent remains. This bias is greater than previous findings would have suggested; diameter measurements above breast height may carry a larger bias than those taken near the bottom of the tree. Often points on the upper stem are farther from the observer, but in steep terrain this is not always so. Whenever possible in this study, the technician located the dendrometer uphill from the tree being measured to get a better look into the crown and to avoid having to move the instrument. Bell and Groman (1968, 1971) found that the dendrometer overestimated smaller diameters by 0.06 inch and larger diameters by 0.04 inch. Presumably, the smaller diameters were those nearer the treetops. But this slight difference can by no means explain the apparent bias discussed here. Furthermore, most volume is in the lower portion of a tree, so inaccuracy in measuring topmost diameters would have a relatively small effect on volume measurement. The explanation offered by Gofas (1967) for overestimating tree volumes with the Bitterlich Spiegelrelaskop does not apply in this case. Unlike the Spiegelrelaskop, which uses a single distance measurement, the Barr and Stroud dendrometer measures slope distance to each point of measurement by the same true coincidence reading as is used in measuring diameter. This slope distance is used to calculate height, thus avoiding Gofas' problem of a diameter measurement taken at a lower height on the tree than that to which it was attributed.

Probably, about half of the 5.4 percent bias that appears in this comparison actually exists; the other half may result from the way in which dendrometer measurements and felled tree measurements were used to calculate tree volume. The NETVSL computer program uses Smalian's formula to compute volume of all tree sections above the stump. Smalian's formula gives reasonably accurate results for segments in the middle portion of the bole, but usually grossly overestimates volume in the butt portion. Dilworth (1961) showed the magnitude of errors that can be expected from Huber's, Smalian's, Rapraeger's, and Sorenson's formulas. His information is summarized in table 3.

Barnes (1945) reported that Smalian's formula gave an overestimate of 10.8 percent for cubic volume in butt logs of western hemlock. Most forest mensuration texts recognize that a truncated paraboloid is not a satisfactory model for the lower portion of a tree stem. Usually, a neiloid is suggested; however, Bruce (1970) has shown that it, too, is inadequate. He had good results with an equation that corresponds to a composite solid comprised of a cylinder with the diameter of the small end and a concave solid of revolution resembling a truncated conoid. Spaeth and others (1952) indicated that overestimates from Smalian's formula may not always be confined to the butt section of trees. In their experiment, Smalian's formula gave volume estimates for fenceposts 7 feet long that averaged 6 percent larger than volumes measured by displacement of water in a tank. Grosenbaugh (1966) emphasized that trees are not simple geometric solids; therefore some error must be expected from formulas based on simple models. He suggested that such error can be kept within acceptable limits by keeping measurement intervals on the stem short enough so that diameter at the small end of a stem section is more than 0.8 the diameter at the large end.

Aside from the way in which cross-sectional stem area was calculated, which caused but little difference, both dendrometer measurements and felled tree measurements were used to calculate volume in exactly the same way. So both sets of volume measurements would be expected to contain some positive bias because of the formula. Bias is not of the same magnitude for both kinds of measurements because stem measurements taken with the dendrometer were generally more widely spaced on the tree stem. Wider spacing of measurements with the dendrometer was caused partly by the observer's inability to see stems through the tree crowns in many places, and partly by his desire to measure as many trees as possible in a day without fully realizing the effect of wider spacing on accuracy.

Table 3.--*Total cubic-foot volume differences: four formulas*[1] *compared with Newton's formula; differences expressed as a percentage of volume computed by Newton's formula. From Dilworth (1961)*

Log group	Formula			
	Huber's	Smalian's	Rapraeger's	Sorenson's
STUDY I (DOUGLAS-FIR)				
Butt	-4.9	+9.9	-0.1	-4.5
Intermediate	-0.8	-1.6	-6.2	-10.6
Top	+0.6	-1.4	-16.3	-22.1
Weighted average	-2.9	+5.8	-4.7	-13.8
STUDY II (WESTERN HEMLOCK)				
Butt	-5.6	+11.2	+0.5	-2.7
Intermediate	+0.5	-1.0	+0.3	-3.7
Top	-2.1	+4.4	-23.0	-28.6
Weighted average	-2.2	+4.4	-1.4	-5.2

[1] Huber's formula: $$V = \frac{\pi}{4 \cdot 144} D_m^2 L$$

Smalian's formula: $$V = \frac{\pi}{4 \cdot 144} \frac{(D_\ell^2 + D_s^2)}{2} L$$

Rapraeger's formula: $$V = \frac{\pi}{4 \cdot 144} \left(D_s + \frac{L}{16}\right)^2 L$$

Sorenson's formula: $$V = \frac{\pi}{4 \cdot 144} \left(D_s + \frac{L}{20}\right)^2 L$$

Newton's formula: $$V = \frac{\pi}{4 \cdot 144} \frac{(D_\ell^2 + 4D_m^2 + D_s^2)}{6} L$$

where

D_s = diameter at the small end of the stem section
D_ℓ = diameter at the large end of the stem section
D_m = diameter at the middle of the stem section
L = section length
V = cubic volume

Young (1967) and Young and others (1967) reported results of an experiment in which volumes of felled trees were calculated with Smalian's formula using stem measurements spaced 8 and 16 feet apart. Volumes regarded as correct were measured by immersing tree pieces in a tank and measuring the amount of water displaced. Table 4 summarizes their results. Spacing measurements 16 feet apart lead to volume measurements that ranged from 3.8 to 15 percent greater than those obtained by a measurement spacing of 8 feet.

In the study reported here, a bias of about 2.4 percent in dendrometer volume measurement was expected because of bias in diameter measurement. An apparent additional bias of 3 percent was found by comparing dendrometer volume measurements with felled tree measurements. Bias probably was introduced by improper use of Smalian's formula in the volume computation program and by spacing dendrometer measurements

Table 4.--*A comparison of tree volumes computed by Smalian's formula with volumes measured by water displacement. From Young and others (1967)*

Species	Computed volume as a fraction of water displacement volume: 8-foot sections	16-foot sections	Ratio
Spruce	0.997	1.035	1.038
Fir	1.032	1.096	1.061
Hemlock	1.035	1.102	1.065
White pine	1.003	1.042	1.039
Cedar	1.072	1.234	1.151

farther apart on the tree trunk than measurements taken after felling. This apparent 3 percent bias is not as great as that shown for most species by Young and others (1967) (table 4) because dendrometer measurements were generally more than 8 feet and less than 16 feet apart.

Cost (1971) supported the conclusion that actual bias in dendrometer volume measurements is little more than would be expected from the 1.19 percent bias in diameter measurement. In his trial of the McClure mirror caliper, two of his observers obtained independent total volume estimates for 25 trees that were within 2.1 percent of the total volume measured after felling. Accuracy was obtained by taking measurements at intervals of only 4 feet in the sawlog portion of the tree and at intervals of 5 feet above the sawlog portion. With measurements so closely spaced, the effect of using an inappropriate geometric model is reduced to an acceptable level.

EVALUATION OF THE BARR AND STROUD DENDROMETER, OTHER OPTICAL INSTRUMENTS, AND FELLED TREE MEASUREMENT

If use of an optical instrument to measure tree volumes is decided upon, the following instruments are available in the United States:

The Barr and Stroud dendrometer, Model F.P. 15;

The Universal Teledendrometer (UTD) (Fricke 1967);

Bitterlich's Relaskop (Bitterlich 1952), Wide Scale Relaskop (Sutter 1965), and Tele-Relaskop, a fairly recent improvement of the relaskop which uses a telescope to magnify the tree image;

Wheeler's penta prism optical caliper (Wheeler 1962); and

The Todis dendrometer.

Two other instruments, not in commercial production but obtainable by special order, are the McClure mirror caliper (McClure 1969) and Mesavage's modified Teletop dendrometer (Mesavage 1969b).

A choice between various instruments (for reasons other than cost) will be based on:

1. Size of trees to be measured

2. Level of accuracy desired, the number of measurements desired, or the purpose for which they are taken

3. Relative productivity of the several measurement techniques in terms of number of trees measured per hour or day, and

4. Restrictions on felling sample trees.

Measurable Trees

The tabulation shows the largest diameter measurements[2] that can be taken with several different instruments:

Barr and Stroud	200 inches
Universal Teledendrometer	24 inches
Bitterlich's wide scale relaskop[3]	±20 feet
Wheeler penta prism caliper	34 inches
McClure mirror caliper	32 inches
Mesavage's modified Teletop	36 inches
Todis dendrometer (direct reading)	31.5 inches
Todis dendrometer (with prisms)	110.2 inches

Although the UTD can measure diameters as large as 24 inches, it must be located exactly 100 feet from the place on the tree stem being measured to do so. This distance requirement severely limits the instrument's usefulness. Mesavage (1969b) and McClure (1969) described ways in which measurement capacities of their instruments could be increased. Unfortunately, the rapidity and simplicity of measurement, advantages of the Teletop and mirror caliper, decrease sharply when extraordinary steps are taken to measure large diameters. In addition, steps involved are likely to cause measurement errors. The Todis dendrometer can measure diameters as large as 31.5 inches without special prisms to make measurement of larger stems possible.

Provision for Height Measurement

If a dendrometer is used to measure standing tree volume, it is necessary to measure not only diameter at various points on the stem but height to points of measurement as well. Only in this way can the length of stem sections be obtained for volume calculation.

Optical Calipers and the Relaskop

Instruments such as the penta prism and mirror calipers measure diameter only. When using these instruments, height must be measured by some independent means. The Forest Resources research unit of the USDA Forest Service in the southeastern States uses sectional aluminum poles, marked at every foot, that stand parallel to the tree bole. Diameter can then be measured with a mirror caliper and the height to that point read from the adjacent pole. The observer is free to move to any position that offers an unobstructed view of the tree. McClure (1968) claimed that under ideal conditions such poles can be extended as much as 120 feet above the ground. Under less than ideal conditions, an extension of 80 feet may represent a more practical limit. Another arrangement involves attachment of a clinometer to the optical caliper. The clinometer is then readily available for measurement of the angle above the horizontal plane. In using the clinometer, it is necessary to measure the distance from the observer's position to the tree, which cannot be done with the caliper.

To measure height with any variant of the Bitterlich relaskop, one must either measure distance from the tree with a tape and use the relaskop to obtain angles above and below the horizontal or use a vertical base of known length beside the tree.

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[2] In some cases larger measurements can be taken by complicating the measurement process.

[3] For the same distance between tree and observer, the conventional relaskop and Tele-Relaskop cannot measure as large a tree as the wide scale relaskop. Since the relaskops are optical forks, one cannot specify an absolute maximum measurement.

Rangefinder Dendrometers

The UTD measures height with an awkward scale arrangement that reproduces angles and distance to the base and upper points on a tree stem. Slope distance from the instrument to the point of measurement is obtained with the Teletop rangefinder.

Mesavage's modified Teletop dendrometer measures slope distance in the same way, but necessary angles are measured with an Abney level or clinometer attached to one side of the dendrometer.

The Barr and Stroud dendrometer is equipped with an inclinometer from which the user reads the sine of the angle formed by the line of sight and the horizontal plane. Distance between the tree stem and the instrument along the line of sight is measured by the dendrometer's rangefinder.

The Todis dendrometer measures the angle to the point of measurement to the nearest 10th of a degree.

Accuracy of Rangefinder Dendrometers

Use of a clinometer or Abney level to measure tree height usually involves measurement of a single distance between the tree and the observer. In heavy brush and on steep slopes, this measurement can be quite inaccurate. Distance to the tree is used with the tangent or cosine of the angle of elevation or depression to compute the height of the point of measurement above or below the horizontal plane. It is assumed that the tree is vertical. If it is not, errors in height measurement will differ in direction and magnitude according to the direction and amount of lean. Inclination of the tree perpendicular to the line of sight will lead to a biased measurement, as will lean parallel to the line of sight. When measuring poles are not used, this method of height measurement must be employed with the penta prism or mirror calipers and with the Bitterlich relaskop.

On the other hand, the Barr and Stroud dendrometer, the UTD, Mesavage's modified Teletop dendrometer, and the Todis dendrometer use the sine of the angle of elevation or depression with the measured slope distance from the instrument to the tree bole at each point of measurement. The tree is assumed to be vertical in this case also, but it can be shown that the bias in height measurement resulting from lean parallel to the line of sight is less than the bias resulting from the clinometer method. With rangefinder dendrometers, the component of a tree's lean that is perpendicular to the line of sight does not affect accuracy of height measurement. The halving line that splits the tree image must be oriented squarely across the stem. For this reason, the angle of elevation is not truly a vertical angle. It is turned in a plane parallel to the tree stem; therefore, it measures the length of the tree rather than height above the horizontal plane. Although most computation techniques do not do so, all measurements necessary to compute stem section length from two distance measurements and an angle are provided by the rangefinder dendrometers. Consequently, lean in either direction is immaterial.

Instrument Readings

Barr and Stroud Dendrometer

The measurements read from the Barr and Stroud dendrometer are the true and false coincidence settings of its rotating prisms. With the use of the manufacturer's tables, charts, or a slide rule approximation (Bruce 1967a; Mesavage 1965; Mesavage 1968), such measurements can be converted to range and diameter in the field. In order to spend all available field time measuring trees, workers do not usually make these conversions. The coincidence readings and sine of the angle of elevation are recorded for each measurement point on each tree and diameters, heights, and volumes are computed in the office. For this reason, coincidence readings are not meaningful to the instrumentman. Consequently, readings that would be questioned if they were read directly as diameters may go unnoticed until it is too late to correct them. However, even though the numbers themselves are meaningless to the observer, a reading that does not conform to the general trend of coincidence readings for a particular tree can often be detected and checked.

Bitterlich's Relaskop

The Bitterlich relaksop and Tele-Relaskop are read in relaskop units, which are proportional to diameter. The constant of proportionality varies for different distances to the tree; so suspicious readings can be detected only by comparison with other readings taken from the same instrument location.

Direct Reading Instruments

Dendrometers based on the Zeiss Teletop, the McClure and Wheeler optical calipers, and the Todis dendrometer are read directly in inches or centimeters for all but large trees. Each reading can be compared immediately to the next lower measurement. The optical calipers can also be used to locate the height at which the tree bole has a given diameter, providing the bole is visible at that point. For example, if 7 inches outside bark is considered the upper limit of merchantability, the instrument can be preset at 7 inches. Then, the user can search up and down the tree until the correct image alinement is obtained. To locate the specified upper limit of merchantability with the Barr and Stroud, several measurements would be required, a trial and error scheme that would be highly impractical.

Tree Stem Image

The McClure mirror caliper, the Wheeler penta prism caliper, and the Bitterlich relaskop are all without image magnification. The Barr and Stroud dendrometer is provided with 5.5 magnification and the Zeiss Teletop, upon which the UTD and Mesavage's modified Teletop dendrometer are based, gives 6 magnification. The Tele-Relaskop offers 8 magnification and the Todis 6.

There are two important advantages to be obtained from image magnification. With magnification, alinement of the half-images for true and false coincidence or reading a relaskop scale can be much more precise. Who can really presume to aline two half-images within one-tenth of an inch at a distance of 200 feet? It is much more probable that this could be done at a distance of 36 feet, which would be the result of 5.5 magnification. If the small bias of the Barr and Stroud in diameter measurement discovered in this study and previously reported by others is indeed due to a characteristic of human vision, magnification could be expected to reduce the effect. Tests of the McClure and Wheeler optical calipers, unsupported by staff or tripod (Robbins and Young 1968), failed to show bias, probably because of a fairly large associated variance in measurements. The optical caliper measurements reported consistently exceeded those of mechanical calipers by about 0.2 inch. Robbins and Young (1968) reported no statistically significant improvement in precision when a 2X lens was attached to the Wheeler caliper. They attributed this to limited magnification and lack of rigid instrument support. Greater magnification and use of a Jacob's staff should increase caliper precision without seriously increasing its weight or hampering its ease of use.

The second advantage of magnification is due to the well-known light gathering properties of an objective lens. On a dull, overcast day, it is often dark under a dense stand of trees. It is even darker when one is looking into tree crowns. Magnification gives a brighter, more sharply defined image than can be obtained by the unaided eye.

On the other hand, ordinary spectacles prevent about 8 percent of the light received from reaching the eye of the wearer (William B. Durbon, O.D., personal communication). It seems likely that two penta prisms would absorb considerably more light; so the right-hand image seen through a penta prism caliper would be far less bright than the tree stem viewed with the naked eye. Under poor light conditions, this could be a serious disadvantage. Light transmission qualities of the mirror caliper are better for the displaced image because two, rather than three, reflections are involved and light does not pass through glass.

Even though the Teletop provides 6 magnification, the writer found in a limited field trial that its image is considerably less bright than that of the Barr and Stroud. In the UTD, the tree image is inverted. Seen through Mesavage's modified Teletop, the image is not inverted, but the right and left sides are transposed. Mesavage (1969b) mentioned these disadvantages in comparing the correctly oriented Barr and Stroud image. Unwanted reflections sometimes appear in the Teletop's field of view and hinder adjustment of false coincidence.

Measurement Time

About the same amount of time is required to take a diameter measurement on the tree bole with the penta prism and mirror calipers, according to Robbins and Young (1968). In his time study of tree measuring instruments, Pfeiffer (1967) found that the time required is about one-third that necessary with the Barr and Stroud dendrometer and the Bitterlich relaskop. This finding makes it appear that more measurements in the same period of time might make up for the lower precision of the two optical calipers. Further, each diameter measurement can be taken from a different position on the ground. It becomes cumbersome to use more than two instrument locations for the same tree when using the Barr and Stroud, the UTD, and the modified Teletop dendrometer.

However, when volume calculation requires that height be recorded at each point of measurement, the rapidity of optical caliper use disappears. So long as measuring poles are not used, a hyposometer of some kind and a distance measurement are needed to measure height. If the observer wishes to make only one measurement of distance, he will have to forego the advantage of being able to use a different instrument location for each measurement. Whether the clinometer is attached to the optical caliper or not, the height measurement will be an entirely separate operation unless both instruments are used with a tripod or Jacob's staff. Without such support the possibility of mistakes exists because the height may be measured to some point on the stem different from the point at which diameter was measured.

The Bitterlich relaskops require either a new distance measurement or a new observation of the vertical reference rod each time the instrument is moved to a new location.

To measure vertical angles, the UTD must be leveled each time it is set up. Leveling is not necessary with the Barr and Stroud or the modified Teletop, because the inclinometer itself is leveled with a bubble at each observation.

Table 5 shows the readings for the seven instruments that must be available for each stem measurement in order to calculate the volume of a tree. Note that unless height measuring poles are used, each instrument requires that at least three items be recorded. Even though distance from the observer to the tree needs to be measured only once so long as instrument location does not change, that measurement can be time consuming and results often inaccurate.

After considering the time study information available in the literature and the number and kind of measurements required for volume calculation, other dendrometers currently in use do not appear to be materially more productive in terms of trees

Table 5.--*Showing the instrument readings which must be recorded at each point of measurement for purposes of volume calculation*[1]

Reading	Barr and Stroud	Wheeler penta prism caliper	McClure mirror caliper	Bitterlich Relaskop[2]	Universal Teledendrometer	Modified Teletop	Todis
Diameter		N	N		N	N	[3]A1
Range (slope distance to stem at point of measurement)					N	N	
Angle of elevation or depression (or a function thereof)	N	A1	A1	A1 A2	N	N	A1 A2
True coincidence reading	N						A2
False coincidence reading	N						A2
Single distance measurement (taped)		A1	A1	A1			
Angle subtended by vertical rod					A2		
Relaskop units					N		
Reading from measuring pole		A2	A2				

[1] N denotes items that are absolutely necessary.

[2] Including the Wide Scale Relaskop and the Tele-Relaskop.

[3] A1,A2 denote items that are necessary under the first or second of two reasonable alternatives.

measured per hour than the Barr and Stroud. Where trees are fairly small, use of a height-measuring pole with mirror or penta prism calipers is an exception to this generalization.

Dendrometer Disadvantages

Low underbrush may not be a serious problem, since measurements at breast height and lower on the tree can be taken with a diameter tape or mechanical calipers. High bushes, the trunks and branches of other trees, and branches of the subject tree itself can seriously affect measurements of the upper portion of a tree's stem. Use of Smalian's formula usually results in overestimates of stem section volume and Huber's formula in underestimates. If stem section length is kept short enough, inadequacy of the geometric model may not introduce a serious bias. In a dense stand or when dealing with dense-crowned species, it is often difficult to keep stem sections short. Also, much time can be spent finding an instrument location that offers the best possible view of the tree stem.

Usually, tree volume inside bark is of primary importance. Measurements with any dendrometer necessarily include the bark and so must be corrected by deducting estimated bark thickness. In most cases, this estimate will be obtained from a predicting equation of one sort or another (Mesavage 1969c; Brickell 1970b). Still, the error and bias in bark thickness estimation will contribute to total error and bias in measurements of tree volume inside bark because even the best regression predictions are subject to some error.

Comparison of Standing Tree Measurement With Felled Tree Measurement

If sample trees can be cut, felling generally is the best practice. On the ground, the entire stem can be measured at any point. Section measurements can be short to assure accuracy of computed volume. Bark thickness can be measured wherever diameter is measured. Furthermore, any desired stem analysis information is available. Dissection of felled sample trees allows calculation of gross, cull, and net volume.

Experience on the Kootenai National Forest has shown that the average assortment of trees (other than ponderosa pine) up to 30 inches d.b.h. can be felled, bucked into 8-foot sections, and measured for gross, cull, and net volume nearly as quickly as they can be measured with the Barr and Stroud dendrometer. Larger trees are more rapidly measured with the dendrometer. Figure 10 illustrates relative rates of felling and dendrometer measurement. Naturally, when cull volume is being measured, it takes longer to measure felled trees with defects than those without. If gross volume only is to be measured, the measurement rate in felled trees will compare even more favorably with that attained by the dendrometer.

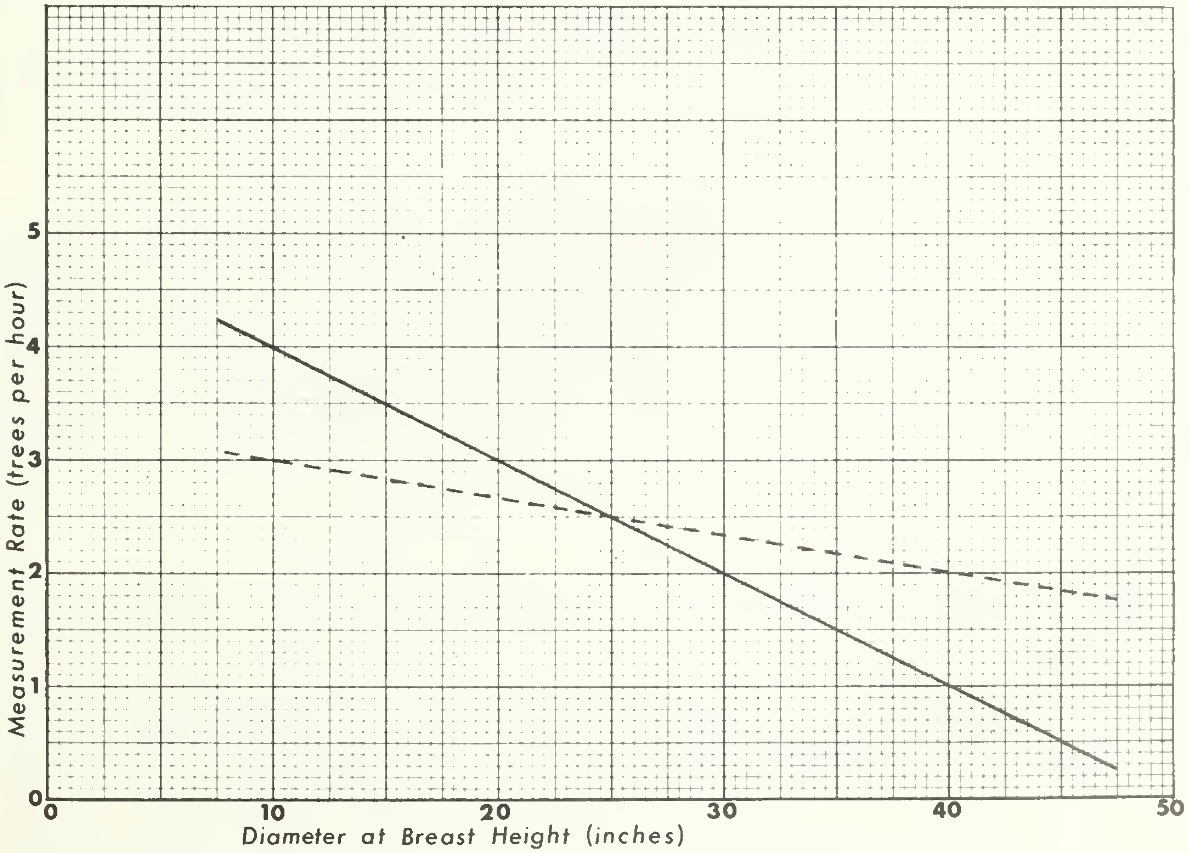

Figure 10.--The general relation of measurement rate to tree size. Felled trees are shown by the solid line; dendrometered trees by the dashed line.

A suitable chain saw and other such necessary equipment as fuel, oil, file, chain wrench, axe, and felling wedges will weigh more than the Barr and Stroud dendrometer, its protective case, and tripod. But, more precautions must be taken to protect the dendromter. Not only is the dendrometer more susceptible to damage than a chain saw, but once damaged or shaken out of adjustment, it will probably have to be sent to Scotland for repairs. On the other hand, chain saw repair shops can be found in almost all forested areas in the United States.

Measurement of Upper Stem Diameters for Purposes Other Than Direct Volume Measurements

Form Class or Form Quotient Estimation

Form class volume tables are often used with an inaccurate estimate of form class. Form class tables are seldom entered with an accurately measured form class for each tree. But, by doing so, Hartman (1967) showed an increase in cruise accuracy on Bureau of Land Management timber sales.

If the accuracy required of a cruise warrants individual form class measurement, use of a dendrometer would be the best way to obtain the upper diameter measurement. For this purpose, the Barr and Stroud, the Todis, and instruments based on the Zeiss Teletop apparently are not as satisfactory as either the mirror caliper or the penta prism caliper. If trees are too large for these instruments, Bitterlich's relaskop could be used. Height measurement is not a concern here because a telescoping pole or one cut in the woods can be used to mark the top of the first log. Aside from their lower cost, advantages of optical calipers for this purpose are that they are lightweight, portable, and are quick and easy to use. Two measurements can be taken from different positions in less time than would be required for one measurement with the Barr and Stroud. Judson (1964) reported that Girard form class can be estimated quite precisely from the ratio of diameter outside bark at the top of the first log to d.b.h. plus double bark thickness at breast height.

Bonner (1968) showed that the ratio of diameter halfway up the tree to d.b.h. in a tree volume prediction equation reduces the standard error of estimate by more than 30 percent. Value of this form quotient was recognized in the last century by Schiffel, Kunze, and Schuberg (Spurr 1952), but instruments for measuring upper diameters were not as precise as those today. Both optical calipers are more satisfactory than the rangefinder dendrometers for measuring diameter at the tree's midpoint. For diameters beyond the capacity of the optical calipers, the Bitterlich relaskop can be used. Stem midpoint can be easily located by averaging slope percent readings taken at the base and tip of a tree with a clinometer. To do this with the sine of the angles would be a more involved and time-consuming process.

Merchantable Length

Locating a specific diameter requires an instrument that can be preset for that diameter. This requirement virtually prohibits practical use of the rangefinder dendrometers unless they are used as optical calipers. To use the relaskop, the desired instrument reading in relaskop units would have to be calculated after measuring horizontal or slope distance to the tree. Distance must be measured in order to measure merchantable stem length; so the required reading for any given distance and diameter limit could be read from a prepared chart. Since the diameter limit of merchantability is usually within the capacity of optical calipers, they would be well-suited to this use.

Growth Measurement

Research plots.--For measuring periodic growth of standing trees, precision at least equal to that of the Barr and Stroud is desirable in any optical instrument. Bias in measurement can be corrected; in the difference between two volume measurements, it might be largely self-correcting. However, a large standard error of the difference would make the periodic growth measurement for a tree unreliable, even if such error represented a small percentage of total volume. When a tree is to be remeasured with the dendrometer, the instrument location should be monumented and the height of the instrument recorded. Future measurements then can be taken at very nearly the same points on the tree by setting up the dendrometer at the same location and height and the inclinometer at the sine of elevation readings previously obtained. Some permanent plot measurement procedures were given by Groman and Berg (1971).

SUMMARY AND SUGGESTIONS

This study disclosed a positive bias of 1.19 percent in diameter measurement with the Barr and Stroud dendrometer. The dendrometer's tendency to slightly overestimate diameters had been reported by others, but bias may be caused by some characteristic of human vision. No bias was found in the measurement of tree heights. Diameter measurement bias could be expected to give an average volume overestimate of about 2.39 percent. A comparison of dendrometered volume with felled tree volume indicated that dendrometer measurements led to an overestimate of about 5.4 percent. This large overestimate may have resulted from using an inadequate geometric model to calculate lower tree stem volume and from wider spacing of dendrometer measurements. If a measurement process is precise, bias can often be removed or minimized by calibrating laboratory instruments.

Random error in volume measurements was found to be about 3 percent of tree volume, much less than the 12 or 15 percent usually associated with volume tables or equations.

For certain tree measurement purposes, the Barr and Stroud dendrometer is potentially a useful instrument. To obtain satisfactory accuracy and precision in field use, the following suggestions are given:

1. A technician assigned to take measurements with the dendrometer should be thoroughly trained in its use and given ample tree measurement experience in the training situation.

2. Any measurement bias peculiar to an instrumentman should be evaluated by a diameter measurement trial such as that described in this paper. If bias can be accurately estimated, it can be removed.

3. Much of the time required to measure a tree is spent finding an instrument location and clearing away brush that obscures the line of sight. The cost of additional stem measurements is small and is a direct investment in accuracy. Therefore, diameter measurements should be no more than 8 feet apart in the tree's lower portion.

4. Use of an adequate geometric model for bole segments is necessary for accuracy, regardless of the way diameter and length measurements are taken. For this reason, any computer programs used for the calculation of tree volumes should be revised, if necessary, so as to use suitable formulas for different portions of the bole.

5. The necessity of discarding a few incorrect observations from the data used in this study emphasizes the requirement of extreme care in reading and recording measurements taken with the Barr and Stroud dendrometer. Instrument readings recorded in the field are meaningless; so erroneous measurements may not be detected when they are taken. Slide rules designed by Bruce (1967a) and Mesavage (1968) can be used to check measurements while work is in progress. However, some of the newer hand-held, battery-powered calculators can be programed, and might be used to convert dendrometer readings to diameters and heights in the field more easily and more accurately than slide rules.

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APPENDIX A

Analysis of Data

The mean of the 87 calipered diameters was 17.68 inches. This figure can be compared with the means of dendrometer measurements shown in table 6 for the nine different (observer X instrument constant) combinations. At most, the means of dendrometer diameters differ from each other by 0.05 inch but they differ from the mean of caliper measurements by about 0.20 inch.

Table 6.--*Means of dendrometer measurements according to observer and instrument constants*

Observer	Instrument constants		
	Grosenbaugh's old set	Grosenbaugh's new set	Direct calibration set
	Inches	*Inches*	*Inches*
P	17.88	17.86	17.88
B	17.88	17.86	17.88
R	17.90	17.88	17.91

The questions to be answered statistically were whether or not the dendrometer measurements differ significantly from caliper measurements on the same tree and, if so, whether or not that difference was the same for all nine sets of dendrometer measurements. The analysis of covariance technique was used for this purpose. At the same time, the regression of dendrometer diameter on caliper diameter can be used to evaluate any bias in dendrometer measurement and the error of dendrometer measurements after correction for bias. If no bias exists in dendrometer measurement, the regression expected is:

$$D_d = b_0 + b_1 D_c + \varepsilon$$

where

D_d = diameter measured with the dendrometer

D_c = diameter measured with calipers

b_0 = 0

b_1 = 1

ε = a random error with mean of zero.

If either or both of the coefficients b_0 and b_1 differ significantly from zero and unity, respectively, then bias is indicated in dendrometer measurements of diameter. The standard error of residuals about the regression provides an estimate of the error of dendrometer measurements after bias has been removed. For use as a standard of comparison in this experiment, caliper measurements of diameter have been considered without error or bias. Freese (1960) has recommended the use of a Chi-square technique for deciding whether or not a new instrument or method of measurement is sufficiently accurate for use when results are compared with those of a known standard. His Chi-square test compares mean square error (variance plus bias) to a predetermined acceptable error. However, even when the Chi-square technique is used in the presence of bias related to size of the standard measurements, the regression of test measurements on the known standard must be estimated if one wishes to separate bias from error. The decision regarding sufficiency of accuracy is then based upon the statistic:

$$\chi^2_{(n-2)\ \mathrm{df}} = \frac{\Sigma\ \mathrm{res}^2}{\sigma^2}$$

where

$\Sigma\ \mathrm{res}^2$ = the sum of squared residuals about regression

σ^2 = a required level of accuracy decided upon through practical considerations.

A computed Chi-square larger than the tabular value for $(n-2)$ degrees of freedom at the desired probability level would indicate insufficient accuracy of the instrument or technique. The regression of measurements being tested on the known standard preserves the distinction between bias of measurements and random error. The purpose of this analysis was to estimate the error and bias of dendrometer measurements, not to decide whether or not their magnitude should preclude use of the instrument. If one can state the level of accuracy his purposes require, the Chi-square test can be easily completed using the residual sum of squares given in this paper.

The analysis of covariance is standard technique for testing differences between several regressions, in this case, whether or not dendrometer bias or lack of bias differs between observers and between different sets of reasonable instrument constants. Appendix B, the format of which was suggested by Freese (1964), shows the results of covariance analysis upon these data. Examination of the variance about the regression showed that variance was nearly proportional to D_c^2, the square of the independent variable. Therefore, all observations were weighted by a value proportional to $1/D_c^2$. The weights were scaled so that the sum of weights was equal to the number of observations. The sums of squares used in this analysis then were weighted sums of squares. The same weights were used for all regressions. It is apparent from appendix B that the variation explained by regression was increased only slightly by allowing each data set to carry its own intercept while forcing a common slope for all sets. An even smaller additional increase in explained variation was obtained by allowing a completely separate regression for each data set. The variance ratios (F statistics) obtained from the differences in residual variation did not even begin to approach the 5 percent significance level. The hypothesis that there is a common regression for three observers and three sets of instrument constants is not rejected. The common regression (appendix B, line 14) is:

$$\hat{D}_d = \hat{b}_0 + \hat{b}_1 D_c$$

where

$\hat{b}_0$ = -0.0024862

$\hat{b}_1$ = 1.0117607

r^2 = 0.998880.

Error statistics for this regression are:

S_{yx} = standard deviation of weighted residuals = 0.19 inch

S_a = standard error of $\hat{b}_0$ = 0.01750789

S_b = standard error of $\hat{b}_1$ = 0.00121211.

The t statistic used to test the difference of the intercept from zero is:

$$t = |\hat{b}_0/S_a| = |-0.0024862/0.01750789| = 0.14200452.$$

For 781 degrees of freedom $t > 1.96$ is necessary to be 95 percent confident of a real difference. The hypothesis that the regression has a zero intercept is retained.

For testing the significance of difference between the regression coefficient and unity, the t statistic is:

$$t = (\hat{b}_1 - 1)/S_b = 0.0117607/0.00121211 = 9.702758.$$

The value of t necessary to be 99.9 percent confident of a real difference is 3.29 for these degrees of freedom. Therefore, the regression coefficient apparently is different from unity.

Because the regression intercept most probably is not different from zero and because nothing inherent in the nature of the data precluded it, a regression constrained to pass through the origin was indicated. Variance of dendrometer measurements around the regression line was proportional to D_c^2 as before; so the regression coefficient was estimated from the weighted uncorrected sums of squares and cross products. With the weights used here, the regression coefficient becomes "the mean of ratios estimator." That is,

$$b_1 = \frac{\Sigma\ (D_d/D_c)}{N} \quad .$$

Simple proof of this can be found in statistical texts (Deming 1928; Draper and Smith 1966; Freese 1964). For the regression, only data compiled with Grosenbaugh's exact instrument constants were used. These constants were slightly, though not significantly, better than the other two sets of constants. Although data obtained through three sets of instrument constants may not constitute independent observations, measurements taken by different observers were independent. This gives the regression:

$$\hat{D}_d = \hat{b}_1 D_c$$

where

$\hat{b}_1$ = 1.0118866

r^2 = 0.99835

S_{yx} = 0.19 inch

S_b = 0.00081

A bias in dendrometer measurements of diameter in excess of 1 percent is indicated.

APPENDIX B

Analysis of Covariance

Regression of dendrometered diameters on calipered diameters; nine data sets (three observers X three sets of instrument constants). Observation weights are proportional to $(1/D_C{}^2)$. Weights are scaled so that sum of weights equals number of observations.

	Set[1]	df	Source	df	Residuals	
					Sum of squares	Mean square
1	PDC	86	Individual regressions within data sets	85	3.1154812	
2	PGO	86		85	3.1178901	
3	PGN	86		85	3.0913137	
4	BDC	86		85	2.9911826	
5	BGO	86		85	2.9136870	
6	BGN	86		85	3.0146680	
7	RDC	86		85	3.1564934	
8	RGO	86		85	3.1911497	
9	RGN	86		85	3.0751897	
10			Total	765	27.6670555	0.03616608
11			Differences for testing slopes	8	.0707543	.00884429
12		774	Common slope, 9 levels	773	27.7378098	.03588332
13			Difference for testing levels	8	.0711811	.00889764
14		782	Common regression	781	27.8089909	.03560691

For test of difference in slopes $F_{8/765} = \frac{0.00884429}{0.03616608} = 0.24454645 \rightarrow$ NS @ 0.05.

For test of difference in levels $F_{8/773} = \frac{0.00889764}{0.03588332} = 0.24796032 \rightarrow$ NS @ 0.05.

[1] Data sets are identified by the observer's initial (P,B,R) and by the instrument constants used in compilation of the data. Thus, PDC means observer P's dendrometer readings compiled with direct calibration instrument constants (see table 1). BGO means observer B's readings compiled with Grosenbaugh's old constants, and RGN means observer R's readings compiled with Grosenbaugh's new constants.

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Brickell, J. E.
1976. Bias and precision of the Barr and Stroud dendrometer under field conditions. USDA For. Serv. Res. Pap. INT-186, 46 p., 83 ref. Intermountain Forest & Range Experiment Station, Ogden, Utah 84401.

Information is presented and methods of testing the Barr and Stroud optical dendrometer in the field are described. Advantages and disadvantages of the Barr and Stroud compared to other means of tree measurement are discussed.

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Headquarters for the Intermountain Forest and Range Experiment Station are in Ogden, Utah. Field programs and research work units are maintained in:

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